Today's learning objective:

By the end of class, I will be able to solve rational and quadratic function problems and interpret curved lines.

Today's language objective:

$$\frac{p(x)}{q(x)}$$

Horizontal asymptote
Vertical asymptote
p(x) and q(x)
Vertex form vs Standard form
Minimum vs Maximum
Axis of Symmetry $x = \frac{-b}{2a}$

} boundaries

$y = a(x-h)^2 + k$

(h, k)

89.) Let $g(x) = 3x - 2$, $h(x) = \dfrac{5x}{x-4}$, $x \neq 4$.

(a) Find an expression for $(h \circ g)(x)$. Simplify your answer.

(b) Solve the equation $(h \circ g)(x) = 0$.

zeroes

$\dfrac{5(3x-2)}{(3x-2)-4} = \dfrac{15x-10}{3x-6}$ (To

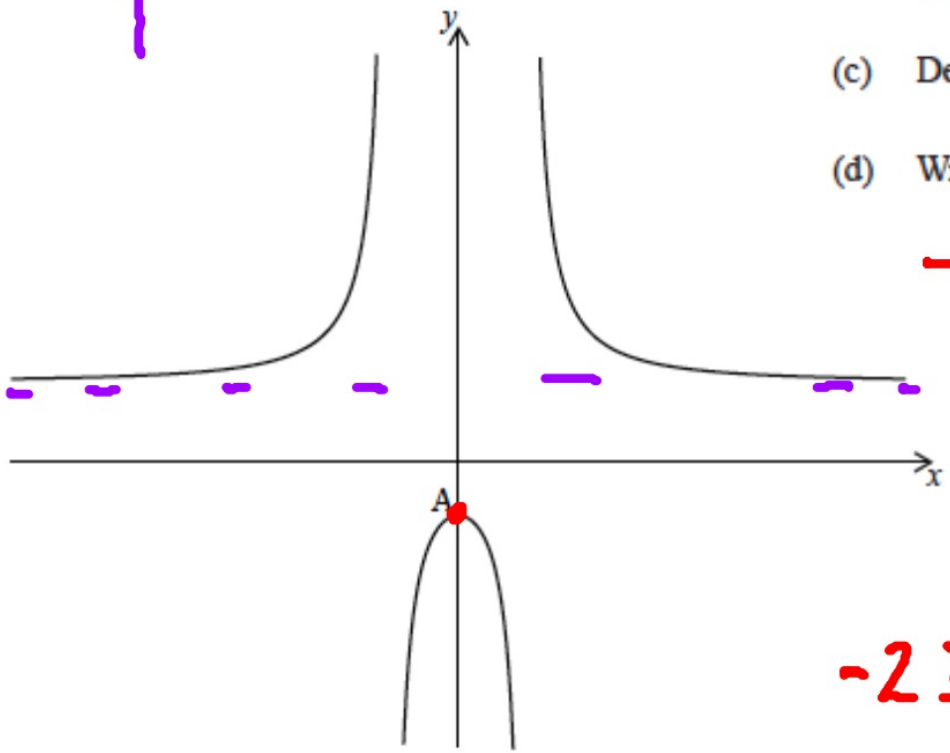
$15x - 10 = 0$

$15x = 10$

v. asympt

$-2($ $=$ $\dfrac{-8x+2}{4x-1} = \dfrac{2(4x-1)}{(4x-1)x = \frac{2}{3}}$

Let $f(x) = 3 + \frac{20}{x^2 - 4}$, for $x \neq \pm 2$. The graph of f is given below.



The y-intercept is at the point A.

(a) (i) Find the coordinates of A.

(ii) Show that $f'(x) = 0$ at A.

(b) The second derivative $f''(x) = \frac{40(3x^2 + 4)}{(x^2 - 4)^3}$. Use this to

(i) justify that the graph of f has a local maximum at A;

(ii) explain why the graph of f does not have a point of i

(c) Describe the behaviour of the graph of f for large $|x|$.

(d) Write down the range of f.

[7 marks]

$f''(x)$

$40(3x^2+4) = 0$

x

$f(x) \to 3$

$-\infty < y \leq -2$

$y = 3$

$3 < y < \infty$

$-2 \geq y > 3$

non-calc

$0 = \frac{0}{16}$

$20(x^2-4)^{-1} \to -20(x^2-4)^{-2} \to$

$A = (0, -2)$

$\frac{-40x}{(x^2-4)^2}$

concavity

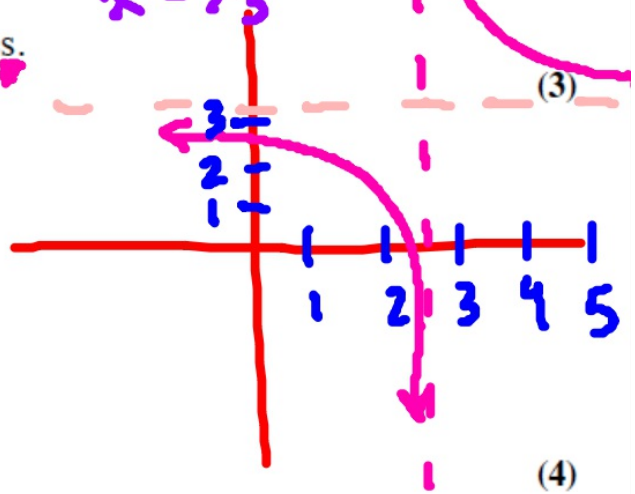
$\frac{160}{-64}$

$\frac{-40(0)}{(0^2-4)^2}$

72.) The function $f(x)$ is defined as $f(x) = 3 + \frac{1}{2x-5}$, $x \neq \frac{5}{2}$.

non-calc

(a) Sketch the curve of f for $-5 \leq x \leq 5$, showing the asymptotes.



(3)

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the x-intercept;

(iii) the value of the y-intercept.

(4)

(c) The region enclosed by the curve of f, the x-axis, and the lines $x = 3$ and $x = a$, is revolved through 360° about the x-axis. Let V be the volume of the solid formed.

(i) Find $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx$.

(ii) Hence, given that $V = \pi\left(\frac{28}{3} + 3\ln 3\right)$, find the value of a.

(10)

Total 17 marks

$$\pi\left[\frac{28}{3} + 3\ln 3\right] = \pi\left| 9x + 3\ln(2x-5) - \frac{1}{4x-10}\right|_{a=4}^{3}$$

$$27 + \cancel{3\cdot 0}\ln 1 - \frac{1}{2}$$

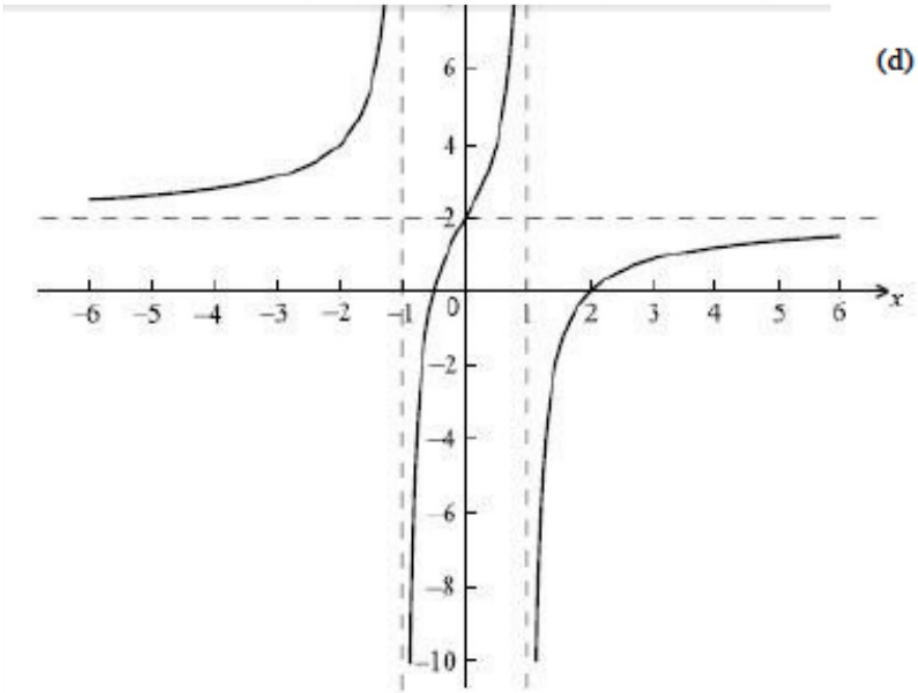
$$\log_e 1 = ?$$

$$e^{?} = 1$$

$$2x-5 = 3$$

73.) Let $f(x) = p - \dfrac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$.

Part of the graph of f, including the asymptot



(a) The equations of the asymptotes are $x = 1$, $x = -1$, $y = 2$. Write down the value of

(i) p;

(ii) q.

(2)

(b) Let R be the region bounded by the graph of f, the x-axis, and the y-axis.

(i) Find the negative x-intercept of f.

(ii) Hence find the volume obtained when R is revolved through 360° about the x-axis.

(c) (i) Show that $f'(x) = \dfrac{3(x^2+1)}{(x^2-1)^2}$.

(ii) Hence, show that there are no maximum or minimum points on

(d) Let $g(x) = f'(x)$. Let A be the area of the region enclosed by the grap between $x = 0$ and $x = a$, where $a > 0$. Given that $A = 2$, find the value

2.) Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k.

(b) The line $y = p$ intersects the graph of f. Find all possible values of p.

(To

Let $f(x) = ax^2 + bx + c$ where a, b and c are rational numbers.

(a) The point P(–4, 3) lies on the curve of f. Show that $16a - 4b + c = 3$.

(b) The points Q(6, 3) and R(–2, –1) also lie on the curve of f. Write down two other linear equations in a, b and c.

) The quadratic equation $kx^2 + (k-3)x + 1 = 0$ has two equal real roots.

(a) Find the possible values of k.

(b) **Write down** the values of k for which $x^2 + (k-3)x + k = 0$ has two equal real roots.

(Total 7

52.) Let $f(x) = 2x^2 + 4x - 6$.

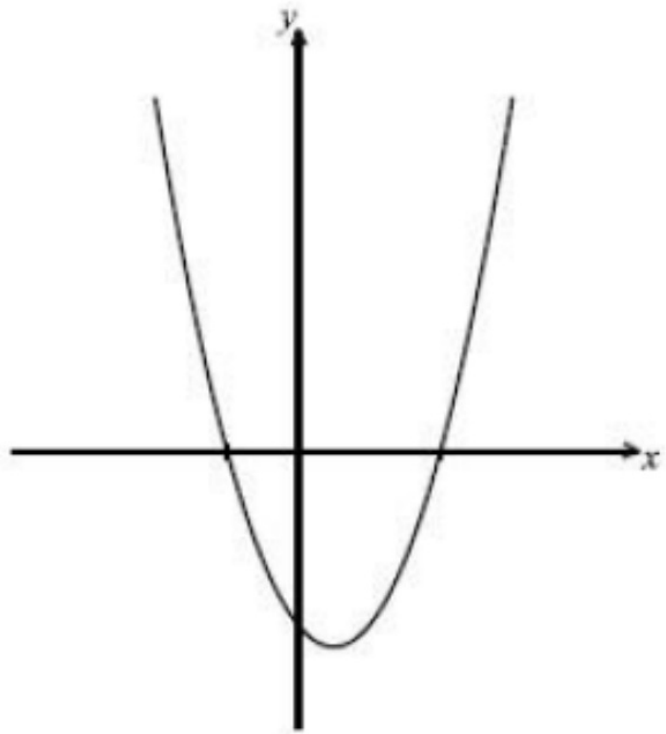
(a) Express $f(x)$ in the form $f(x) = 2(x-h)^2 + k$.

(b) Write down the equation of the axis of symmetry of the graph of f.

(c) Express $f(x)$ in the form $f(x) = 2(x-p)(x-q)$.

(Total 6

45.) The following diagram shows part of the graph of f, where $f(x) = x^2 - x - 2$.



(a) Find both x-intercepts.

(b) Find the x-coordinate of the vertex.

(Total 6 r

58.) Let $f(x) = 2x^2 - 12x + 5$.

(a) Express $f(x)$ in the form $f(x) = 2(x-h)^2 - k$.

(b) Write down the vertex of the graph of f.

(c) Write down the equation of the axis of symmetry of the graph of f.

(d) Find the y-intercept of the graph of f.

(e) The x-intercepts of f can be written as $\frac{p \pm \sqrt{q}}{r}$, where $p, q, r \in \mathbb{Z}$.

Find the value of p, of q, and of r.

(Total 15