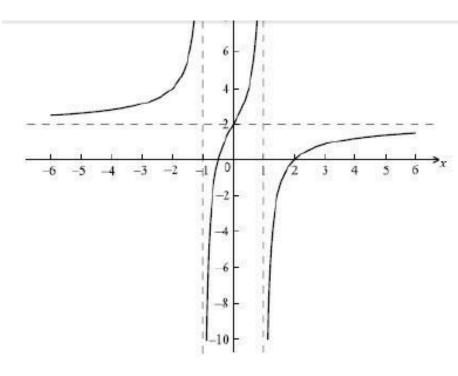
(7)

ASSIGNMENT: Rational Functions

[SL calc]

73.) Let
$$f(x) = p - \frac{3x}{x^2 - q^2}$$
, where $p, q \in \mathbb{R}^+$.

Part of the graph of f, including the asymptotes, is shown below.



- (a) The equations of the asymptotes are x = 1, x = -1, y = 2. Write down the value of
 - (i) p;
 - (ii) q. (2)
- (b) Let R be the region bounded by the graph of f, the x-axis, and the y-axis.
 - Find the negative x-intercept of f.
 - (ii) Hence find the volume obtained when R is revolved through 360° about the x-axis.
 (7)
- (e) (i) Show that $f'(x) = \frac{3(x^2+1)}{(x^2-1)^2}$.
 - (ii) Hence, show that there are no maximum or minimum points on the graph of f.

 (8)
- (d) Let g(x) = f'(x). Let A be the area of the region enclosed by the graph of g and the x-axis, between x = 0 and x = a, where a > 0. Given that A = 2, find the value of a.

(a) (i) p = 2 A1 N1

(ii)
$$q = 1$$
 A1 N1

(b) (i)
$$f(x) = 0$$
 (M1)
$$2 - \frac{3x}{x^2 - 1} = 0$$
 $(2x^2 - 3x - 2 = 0)$ A1

$$x^2 - 1 = 0$$
 $(2x - 3x - 2 = 0)$

$$x = -\frac{1}{2} x = 2$$

$$\left(-\frac{1}{2},0\right)$$
 A1 N2

(ii) Using
$$V = \int_a^b \pi y^2 dx$$
 (limits not required) (M1)

$$V = \int_{\frac{1}{2}}^{0} \pi \left(2 - \frac{3x}{x^2 - 1} \right)^2 dx$$
 A2

$$V = 2.52$$
 A1 N2

eg Product or quotient rule

Correct derivatives of
$$3x$$
 and $x^2 - 1$ A1A1

$$eg \frac{-3(x^2-1)-(-3x)(2x)}{(x^2-1)^2}$$

$$f'(x) = \frac{-3x^2 + 3 + 6x^2}{(x^2 - 1)^2}$$
 A1

$$f'(x) = \frac{3x^2 + 3}{(x^2 - 1)^2} = \frac{3(x^2 + 1)}{(x^2 - 1)^2}$$
 AG NO

(ii) METHOD 1

Evidence of using
$$f'(x) = 0$$
 at max/min (M1)

$$3(x^2+1) = 0(3x^2+3=0)$$
 A1

METHOD 2

Evidence of using
$$f'(x) = 0$$
 at max/min (M1)

Sketch of
$$f'(x)$$
 with good asymptotic behaviour A1

NAME:

DATE: 0/18A-19B/2018

METHOD 3

Evidence of using
$$f'(x) = 0$$
 at max/min (M1)

Evidence of considering the sign of f'(x) A1

$$f'(x)$$
 is an increasing function $(f'(x) > 0$, always)

Therefore, no maximum or minimum. AG NO

Area =
$$\int_0^a g(x) dx \left(\text{or } \int_0^a f'(x) dx \text{ or } \int_0^a \frac{3x^2 + 3}{(x^2 - 1)^2} dx \right)$$
 A1

Recognizing that
$$\int_0^a g(x) dx = f(x) \Big|_0^a$$
 A2

Correct equation A1

$$eg \int_0^a \frac{3x^2 + 3}{(x^2 - 1)^2} dx = 2, \left[2 - \frac{3a}{a^2 - 1}\right] - \left[2 - 0\right] = 2, 2a^2 + 3a - 2 = 0$$

$$a = \frac{1}{2} \qquad a = -2$$