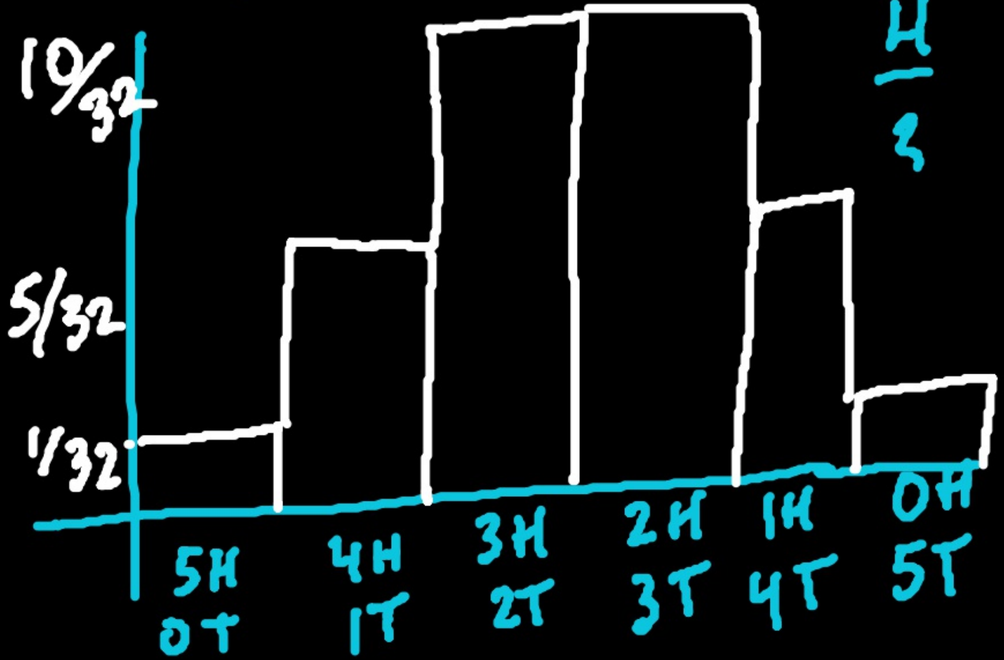
We're going to flip an unbiased coin five times.

What do you expect to happen? $10 = \frac{5!}{3!(5-3)!} = C$

Perform the experiment and record data.

Write your results on the board.



H
3

T
1
2
3
1
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Today's learning objective:

By the end of class, I will be able to identify binomial probability distributions and solve interesting problems with bell curve probability.

Today's language objective:

Binomial probability distribution
Bell Curve
Normal Curve

exact
p = pdf = probability (=)

c = cdf = cumulative (at least)

- 2 outcomes
- Probability
- !
- Headache (combos)

3 Heads

prob.

combinations

5.8	Binomial distribution	$X \sim \mathrm{B}(n, p) \quad \Rightarrow \quad \mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, \; r = 0, 1, \ldots, n$

2 outcomes

probability

$$P(X=3) = \begin{bmatrix} 5 \\ 3 \end{bmatrix} p^3 (1-p)^2$$

$$P(X=3) = \begin{bmatrix} 5 \\ 3 \end{bmatrix} 0.5^3 (1-0.5)^2$$

$$\frac{10}{32} = 10 \cdot 0.5^3 \cdot 0.5^2$$

5.) A box holds 240 eggs. The probability that an egg is brown is 0.05.

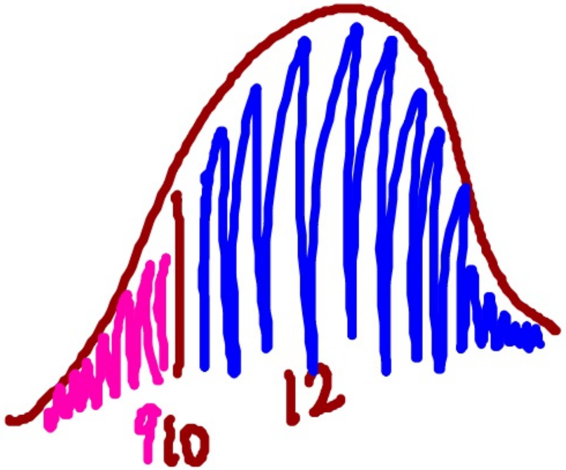
(a) Find the expected number of brown eggs in the box. = 12
(2)

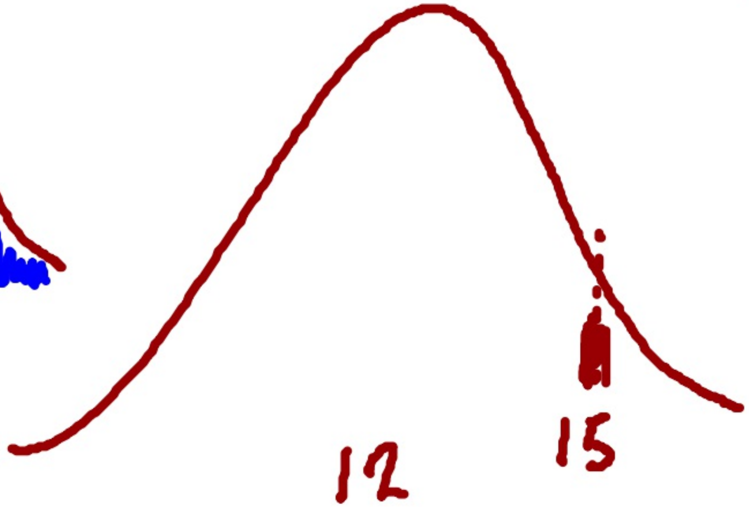
(b) Find the probability that there are 15 brown eggs in the box. .0733
(2)

(c) Find the probability that there are at least 10 brown eggs in the box.
(3)

(Total 7 marks)

$P(X \geq 10) = .764$





24.) A multiple choice test consists of ten questions. Each question has five answers. Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a) Find the expected number of questions Jose answers correctly. **(1)**

$.2 \cdot 10 =$ 2

(b) Find the probability that Jose answers exactly three questions correctly. **(2)**

$P(x=3) = \binom{10}{3} .2^3 (1-.2)^7$ 0.201

(c) Find the probability that Jose answers more than three questions correctly. **(3)**

(Total 6 marks)

$P(x>3) =$ $\boxed{0.121}$

$1 - P(x \leq 3) = 0.121$

11.) Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games.

(2)

For game B, the probability that Evan wins is p. He plays game B seven times.

(b) Write down an expression, in terms of p, for the probability that he wins exactly four games.

$P(X=4) = \begin{bmatrix} 7 \\ 4 \end{bmatrix} p^4 (1-p)^3$

(2)

(c) Hence, find the values of p such that the probability that he wins exactly four games is 0.15.

$.15 = \begin{bmatrix} 7 \\ 4 \end{bmatrix} p^4 (1-p)^3$

(3)

(Total 7 marks)

5.8	Binomial distribution	$X \sim \mathrm{B}(n, p) \Rightarrow \mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r},\ r = 0, 1, \ldots, n$

$.15 = 35p^4(1-p)^3 - .15 = 0$

$-.15$

$p = 0.356,$
0.770

13.) Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

(a) Jan tosses the two dice once. Find the probability that she wins a prize.

(3)

$1/9$

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes.

(2)

(Total 5 marks)

$P(X=3) = \boxed{.0426}$

4	1	2	3
1	4	3	2

$1/36$

$= \frac{4}{36} = \frac{1}{9}$

works .96

28.) A factory makes switches. The probability that a switch is defective is 0.04.

The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample.

(2)

(b) Find the probability that there are exactly six defective switches in the sample.

$$P(x=6) = \binom{100}{6} .04^{6} (1-.04)^{94} = \boxed{.105}$$

(2)

(c) Find the probability that there is at least one defective switch in the sample.

(3)

(Total 7 marks)

$$1 - P(x=0) = .983$$

32.) Paula goes to work three days a week. On any day, the probability that she goes on a red bus is $\frac{1}{4}$.

(a) Write down the expected number of times that Paula goes to work on a red bus in one week.

$.75$

(2)

In one week, find the probability that she goes to work on a red bus

(b) on exactly two days;

$P(x=2) = \binom{3}{2} .25^2 (1-.25)$

$= .141$

(2)

(c) on at least one day.

(3)

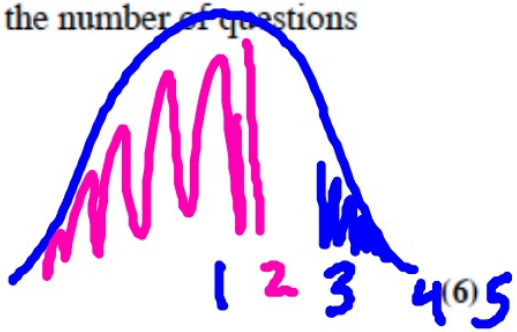
(Total 7 marks)

1, 2, 3

$1 - P(x=0) = .578$

15.) A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Mark answers correctly.



(a) (i) Find $E(X)$.

(ii) Find the probability that Mark passes the test.

(6)

Bill also takes the test. Let Y be the number of questions that Bill answers correctly.
The following table is the probability distribution for Y.

.03 .12 .04

y	0	1	2	3	4	5
P$(Y = y)$	0.67	0.05	$a + 2b$	$a - b$	$2a + b$	0.04

(b) (i) Show that $4a + 2b = 0.24$.

$.24 = 4a + 2b$

(ii) Given that $E(Y) = 1$, find a and b.

$.75 = 13a + 5b$

$-.6 = -10a - 5b$

$\boxed{P = .19}$

(8)

(c) Find which student is more likely to pass the test.

$.15 = 3a$

$a = .05$

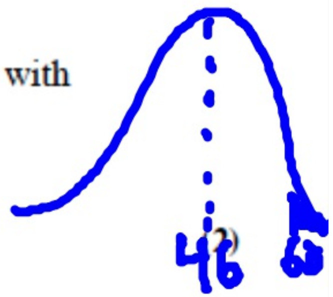
(3)

(Total 17 marks)

21.) A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.



(a) For Route A, find the probability that the journey takes **more** than 60 minutes.

.0807

(2)

(b) For Route B, the probability that the journey takes **less** than 60 minutes is 0.85.
Find the value of μ.

(3)

(c) The van sets out at 06:00 and needs to arrive before 07:00.

(i) Which route should it take?

(ii) Justify your answer.

(3)

(d) On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that

(i) it arrives before 07:00 on all five days; .444

(ii) it arrives before 07:00 on at least three days. .973

(5)
(Total 13 marks)