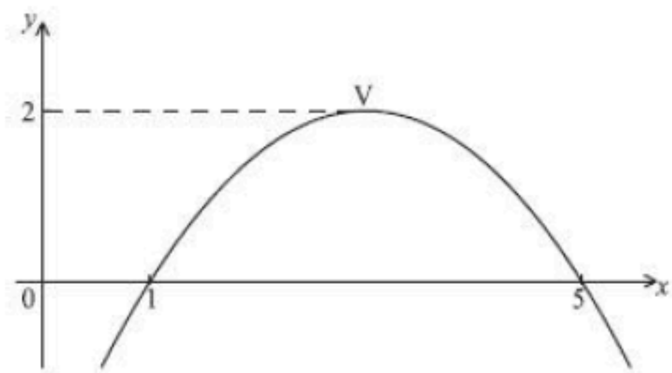
87.) Part of the graph of the function $y = d(x - m)^2 + p$ is given in the diagram below.

The x-intercepts are $(1, 0)$ and $(5, 0)$. The vertex is $V(m, 2)$.



[non-calc]

(a) Write down the value of

(i) m;

(ii) p.

(b) Find d.

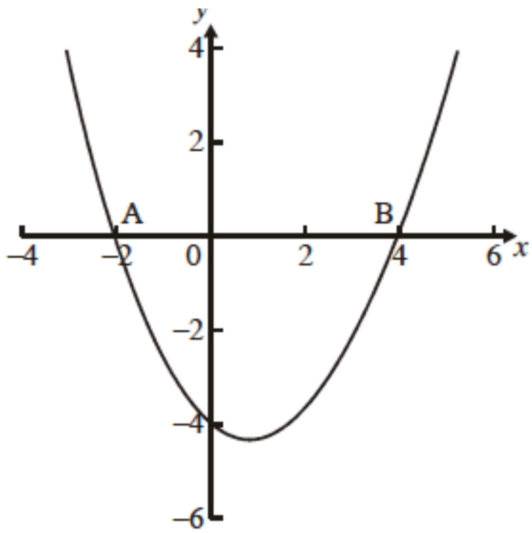
(Total 6 marks)

172.) Let $f(x) = \sqrt{x}$, and $g(x) = 2^x$. Solve the equation

$(f^{-1} \circ g)(x) = 0.25$.

[non-calc]

99.) The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram below.



[non-calc]

(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$.

(5)

(b) (i) Find $\frac{dy}{dx}$.

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P.

(4)

(c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.

(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again.

167.) The function f is given by

$$f(x) = \frac{2x+1}{x-3}, x \in \mathbb{R}, x \neq 3.$$

[non-calc]

(a) (i) Show that $y = 2$ is an asymptote of the graph of $y = f(x)$. **(2)**

(ii) Find the vertical asymptote of the graph. **(1)**

(iii) Write down the coordinates of the point P at which the asymptotes intersect. **(1)**

(b) Find the points of intersection of the graph and the axes. **(4)**

(c) Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines. **(4)**

(d) Show that $f'(x) = \frac{-7}{(x-3)^2}$ and hence find the equation of the tangent at the point S where $x = 4$. **(6)**

(e) The tangent at the point T on the graph is parallel to the tangent at S.

Find the coordinates of T. **(5)**

(f) Show that P is the midpoint of $[ST]$.

11.) (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$;

(ii) $\log_c 25$.

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

[non-calc]

12) [non-calc]

[Maximum mark: 15]

A school collects cans for recycling to raise money. Sam's class has 20 students.
The number of cans collected by each student in Sam's class is shown in the following stem and leaf diagram.

Stem	Leaf
2	0, 1, 4, 9, 9
3	1, 7, 7, 7, 8, 8
4	1, 2, 2, 3, 5, 6, 7, 8
5	0

Key: 3|1 represents 31 cans

(a) Find the median number of cans collected. [2]

The following box-and-whisker plot also displays the number of cans collected by students in Sam's class.

(b) (i) Write down the value of a.

(ii) The interquartile range is 14. Find the value of b. [3]

Answer key

87.) (a) (i) $m = 3$ A2 N2

(ii) $p = 2$ A2 N2

(b) Appropriate substitution M1

$eg\ 0 = d(1-3)^2 + 2, 0 = d(5-3)^2 + 2, 2 = d(3-1)(3-5)$

$d = -\frac{1}{2}$ A1 N1

[6]

172.) $x = g^{-1}(f(0.25))$ (M1)

$= \log_2((0.25)^{1/2})$ (A1)

$= \log_2\left(\frac{1}{2}\right)$ (A1)

$= -1$ (A1)

OR

$f^{-1}(x) = x^2$ (M1)

$= (f^{-1} \circ g)(x) = f^{-1}(2^x) = 2^{2x}$ (M1)

Therefore, $2^{2x} = 0.25 = 2^{-2}$ (M1)

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$ (A1) (C4)

[4]

99.) (a) (i) $p=-2 \quad q=4$ (or $p=4, q=-2$) (A1)(A1) (N1)(N1)

(ii) $y=a(x+2)(x-4)$

$8=a(6+2)(6-4)$ (M1)

$8=16a$

$a=\frac{1}{2}$ (A1) (N1)

(iii) $y=\frac{1}{2}(x+2)(x-4)$

$y=\frac{1}{2}(x^2-2x-8)$

$y=\frac{1}{2}x^2-x-4$ (A1) (N1)5

(b) (i) $\frac{\mathrm{d}y}{\mathrm{d}x}=x-1$ (A1) (N1)

(ii) $x-1=7$ (M1)

$x=8, y=20$ (P is $(8, 20)$) (A1)(A1) (N2)4

(c) (i)when $x = 4$, gradient of tangent is $4 - 1 = 3$ (may be implied)(A1)

gradient of normal is $-\frac{1}{3}$ (A1)

$y-0=-\frac{1}{3}(x-4)$ $\left(y=-\frac{1}{3}x+\frac{4}{3}\right)$ (A1) (N3)

(ii) $\frac{1}{2}x^2-x-4=-\frac{1}{3}x+\frac{4}{3}$ (or sketch/graph) (M1)

$\frac{1}{2}x^2-\frac{2}{3}x-\frac{16}{3}=0$

$3x^2-4x-32=0$ (may be implied) (A1)

$(3x+8)(x-4)=0$

$x=-\frac{8}{3}$ or $x=4$

$x=-\frac{8}{3}$ (-2.67) (A1) (N2)6

[15]

167.) (a) (i) $f(x) = \dfrac{2x+1}{x-3}$

$= 2 + \dfrac{7}{x-3}$ by division or otherwise (M1)

Therefore as $|x| \to \infty$ $f(x) \to 2$ (A1)

$\Rightarrow y = 2$ is an asymptote (AG)

OR $\lim_{x\to\infty} \dfrac{2x+1}{x-3} = 2$ (M1)(A1)

$\Rightarrow y = 2$ is an asymptote (AG)

OR make x the subject

$yx - 3y = 2x + 1$

$x(y-2) = 1 + 3y$ (M1)

$x = \dfrac{1+3y}{y-2}$ (A1)

$\Rightarrow y = 2$ is an asymptote (AG)

***Note:** Accept inexact methods based on the ratio of the coefficients of x.*

(ii) Asymptote at $x = 3$ (A1)

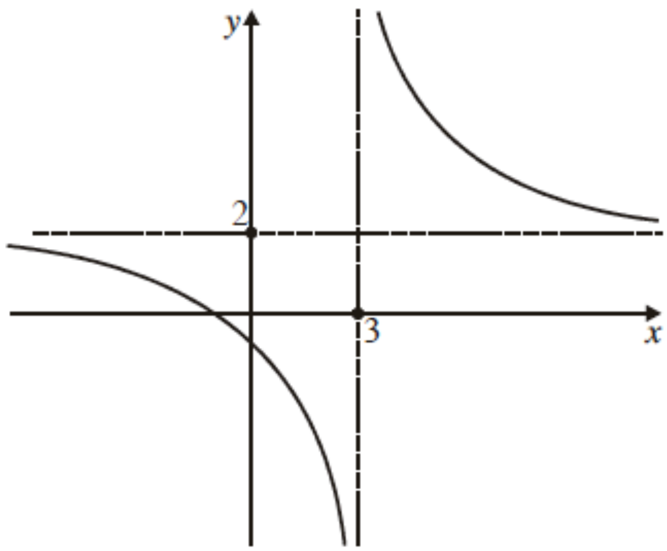
(iii) $P(3, 2)$ (A1) 4

(b) $f(x) = 0 \Rightarrow x = -\dfrac{1}{2}\left(-\dfrac{1}{2}, 0\right)$ (M1)(A1)

$x = 0 \Rightarrow f(x) = -\frac{1}{3}\ \left(0, -\frac{1}{3}\right)$ (M1)(A1) 4

***Note:** These do not have to be in coordinate form.*

(c)



(A4) 4

Note: *Asymptotes (A1)*
Intercepts (A1)
"Shape" (A2).

(d) $f'(x) = \dfrac{(x-3)(2)-(2x+1)}{(x-3)^2}$ (M1)

$= \dfrac{-7}{(x-3)^2}$ (A1)

= Slope at any point

Therefore slope when $x = 4$ is -7 (A1)
And $f(4) = 9$ ie $S(4, 9)$ (A1)
$\Rightarrow$ Equation of tangent: $y - 9 = -7(x - 4)$ (M1)
$7x + y - 37 = 0$ (A1) 6

(e) at T, $\dfrac{-7}{(x-3)^2} = -7$ (M1)

$\Rightarrow (x-3)^2 = 1$ (A1)
$x - 3 = \pm 1$ (A1)

$\left.\begin{array}{l} x = 4 \text{ or } 2 \\ y = 9 \text{ or } -5 \end{array}\right\} \begin{array}{l} S(4,9) \\ T(2,-5) \end{array}$ (A1)(A1) 5

(f) Midpoint $[ST] = \left(\dfrac{4+2}{2}, \dfrac{9-5}{2}\right)$

$= (3, 2)$
= point P (A1) 1

[24]