Adjusted for inflation, the average annual return of the S&P 500 for the last 86 years is 7.7%.

If you could invest $100 into the S&P 500 right now and leave it there for ~~50~~ years, should you do it?

40

$U_n = u_1 r^{(n-1)}$

\$3,789

\$1,804

Today's learning objective:

By the end of class, I will be able to solve arithmetic sequences and series problems.

Today's language objective:

Arithmetic vs Geometric
d =
Sequence vs Series
I will utilize this vocabulary in cooperative groups.

1.1	The n^{th} term of an arithmetic sequence	$u_n = \underbrace{u_1 + (n-1)d}$
	The sum of n terms of an arithmetic sequence	$S_n = \underbrace{\frac{n}{2}(2u_1 + (n-1)d)} = \underbrace{\frac{n}{2}(u_1 + u_n)}$

1.) In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.

non-calc

(a) Find d.

8 = 2 + (3–1)d; 6 = 2d; d = 3

(b) Find u_{20}.

u20 = 2 + (20–1)(3) = 59

(c) Find S_{20}.

S20 = 20/2(2*2 + (20–1)3) = 610

(Total 6 mark

calc

2.) In an arithmetic sequence $u_1 = 7$, $u_{20} = 64$ and $u_n = 3709$.

64 = 7 + (20–1)d; 57 = 19d

(a) Find the value of the common difference.

d = 3

(b) Find the value of n.

3709 = 7 + (n–1)3

3702 = 3n – 3

3705 = 3n; n = 1,235

(Total 5 mark

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

3.) Consider the arithmetic sequence 3, 9, 15, ..., 1353.

calc

(a) Write down the common difference.

d = 6

(b) Find the number of terms in the sequence.

1353 = 3 + (n–1)6

1350 = 6n – 6

1356 = 6n; n = 226

(c) Find the sum of the sequence.

(Total 6 mar

S226 = 226/2(2*3 + (226–1)6) = 153,22

4.) An arithmetic sequence, $u_1, u_2, u_3, \ldots$, has $d = 11$ and $u_{27} = 263$

calc

(a) Find u_1.

263 = u1 + (27–1)11; 263 = u1 + 286

u1 = –23

(b) (i) Given that $u_n = 516$, find the value of n.

(ii) For this value of n, find S_n.

516 = –23 + (n–1)11

539 = 11n – 11

550 = 11n; n = 50

S = 50/2(–23 + 516) = 12,325

(Total 6 mar

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

5,500 ~~30~~ 50

44.) ~~$1000~~ is invested at the beginning of each year for ~~10~~ years.

The rate of interest is fixed at 7.5% per annum. Interest is compounded annually.

Calculate, giving your answers to the nearest dollar

(a) how much the first $1000 is worth at the end of the ~~ten years~~ $1,917

(b) the total value of the investments at the end of the ten years.

$14,147

x .15

The n^{th} term of a geometric sequence	$u_n = u_1 r^{n-1}$
The sum of n terms of a finite geometric sequence	$S_n = \dfrac{u_1(r^n - 1)}{r - 1} = \dfrac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$
The sum of an infinite geometric sequence	$S_\infty = \dfrac{u_1}{1 - r}$, $\lvert r \rvert < 1$

$1.25mm

.22

$1.03 mm

$$-6$$

$$2\tan^2 x + \frac{3}{\cos x} = 0 \quad -\tfrac{1}{2}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cancel{\cos x = 2}$$

$$\frac{1}{\cos x} = \frac{1}{2}$$

$$2\tan^2 x + 3\sec x = 0$$

$$0 < x < 2\pi \qquad 2(\sec^2 x - 1) + 3\sec x = 0$$

$$\boxed{\frac{1}{\cos x} = -2}$$

$$\sec x = \frac{1}{2}$$

$$\cos x = \frac{-1}{2}$$

$$2\sec^2 x + 3\sec x - 2$$

$$\sec x = -2$$

$$2x^2 + 3x - 2 = 0$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\begin{matrix} 2x & -1 \\ x & 2 \end{matrix}$$

$$2\sec x = 1$$

$$\sec x = -2$$

6.) The n^{th} term of an arithmetic sequence is given by $u_n = 5 + 2n$. calc

(a) Write down the common difference.

u1 = 7; u2 = 9; d = 2

(1)

(b) (i) Given that the n^{th} term of this sequence is 115, find the value of n.

(ii) For this value of n, find the sum of the sequence.

(5)

(Total 6 marks)

115 = 7 + (n–1)2

108 = 2n – 2; n = 55; S55 = 55/2(7+115) = 3,355

7.) In an arithmetic series, the first term is –7 and the sum of the first 20 terms is 620. calc

(a) Find the common difference.

620 = 20/2(2*-7 + (20-1)d)

620 = 10(–14 + 19d)

(3)

62 = –14 + 19d

76 = 19d; d = 4

(b) Find the value of the 78^{th} term.

u78 = –7 + (78–1)4 = 301

(2)

(Total 5 marks)

erm of an ic sequence	$u_n = u_1 + (n-1)d$
of n terms of an ic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

10.) In an arithmetic sequence, $S_{40} = 1900$ and $u_{40} = 106$. Find the value of u_1 and of d.

(Total 6 marks)

106 = u1 + (40–1)d; 106 = –11 + 39d calc

106 = u1 + 39d 117 = 39d; d=3

1900 = 40/2 (u1 + 106)

1900 = 20u1 + 2120; –220 = 20u1; u1 = –11

11.) Consider the arithmetic sequence 2, 5, 8, 11, ...

(a) Find u_{101}. d = 3; u101 = 2 + (101–1)3; u101 = 302

(3)

(b) Find the value of n so that $u_n = 152$.

152 = 2 + (n–1)3

150 = 3n – 3

153 = 3n; n = 51

(3)

(Total 6 marks)

calc

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

14.) In an arithmetic sequence $u_{21} = -37$ and $u_4 = -3$.

(a) Find

$d = -2$

(i) the common difference;

(ii) the first term. $u1 = 3$

(b) Find S_{10}.

$S10 = 10/2(2*3 + (10-1)*-2)$

$S10 = 5(6 - 18)$

$S10 = -60$

(Total 7 m

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

20.) (a) Write down the first three terms of the sequence $u_n = 3n$, for $n \geq 1$.

(1)

(b) Find

(i) $\sum_{n=1}^{20} 3n$;

(ii) $\sum_{n=21}^{100} 3n$.

(5)
(Total 6 marks)

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

27.) Let S_n be the sum of the first n terms of an arithmetic sequence, whose first three terms are u_1, u_2 and u_3. It is known that $S_1 = 7$, and $S_2 = 18$.

(a) Write down u_1.

(b) Calculate the common difference of the sequence.

(c) Calculate u_4.

31.) Gwendolyn added the multiples of 3, from 3 to 3750 and found that

$$3 + 6 + 9 + \ldots + 3750 = s.$$

Calculate s.

$u_n = 3750 = 3 + 3n - 3$

$3750 = 3n$

$1250 = n$

$S_{1250} = 625(3 + 3750)$

$625(3753)$

$\boxed{S_{1250} = 2{,}345{,}625}$

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$