

$$4x + 2$$

$$f'(x) = 4$$

$$4x + C$$

$$\int 4 dx$$

indefinite  
integral

$$2x^2 + 6$$

$$f'(x) = 4x$$

$$2x^2 + C$$



$$\int 4x dx$$

Why do we add "+ C" on the end of indefinite integrals?

## Today's learning objective:

By the end of class, I will be able to calculate the value of "C" with the integral by utilizing additional information in problems.

## Today's language objective:

Constant

7

Integral

Let  $f'(x) = 12x^2 - 2$ .

Given that  $f(-1) = 1$ , find  $f(x)$ .

(Tot

$$f(x) = 4x^3 - 2x + 3 \quad \int (12x^2 - 2) dx$$

$$4(-1)^3 - 2(-1) + C = 1$$

$$4(-1) + 2 + C = 1$$

$$-4 + 2 + C = 1$$

$$C = 3$$

$$\frac{12x^{2+1}}{3} - \frac{2x^1}{1} + C$$

$$4x^3 - 2x + C = f(x)$$

The curve  $y = f(x)$  passes through the point  $(2, 6)$ .

Given that  $\frac{dy}{dx} = 3x^2 - 5$ , find  $y$  in terms of  $x$ .

$$\int (3x^2 - 5) dx$$

↓

$$x^3 - 5x + C$$

$$6 = (2)^3 - 5(2) + C$$

$$6 = 8 - 10 + C$$

$$C = 8$$

$$y = x^3 - 5x + 8$$

It is given that  $\frac{dy}{dx} = x^3 + 2x - 1$  and that  $y = 13$  when  $x = 2$ .

Find  $y$  in terms of  $x$ .

*Working:*

$$\int (x^3 + 2x - 1) dx$$

$$\downarrow$$
$$\frac{x^4}{4} + x^2 - x + C$$

$$13 = \frac{(2)^4}{4} + (2)^2 - (2) + C$$

$$13 = 6 + C$$
$$7 = C$$

$$y = \frac{x^4}{4} + x^2 - x + 7$$

*Answer:*

(Tot

The derivative of the function  $f$  is given by  $f'(x) = \frac{1}{1+x} - 0.5 \sin x$ , for  $x \neq -1$ .

The graph of  $f$  passes through the point  $(0, 2)$ . Find an expression for  $f(x)$ .

$$y = x$$

Working:

$$\ln = \log_e$$

$$\log_e 1 = ?$$

$$e^? = 1$$

$$f(x) = \ln(1+x) + 0.5 \cos x + C$$

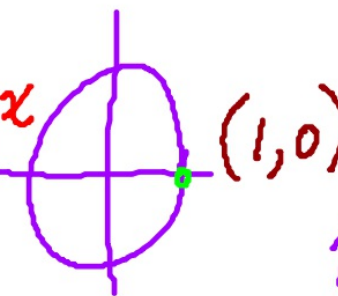
$$2 = \ln(1+0) + 0.5 \cos 0 + C$$

$$2 = \frac{\ln 1}{0} + 0.5(1) + C$$

Answer:

$$f(x) = \ln(1+x) + 0.5 \cos x + 1.5$$

non-calc

$$\frac{1,000,000}{500} = \frac{500(1.07)^x}{500}$$


$$\log_{1.07} 2000 = x$$

(Total 6 marks)

$$2000 = 1.07^x$$



Let  $f'(x) = 1 - x^2$ . Given that  $f(3) = 0$ , find  $f(x)$ .

*Working:*

$$f(x) = x - \frac{x^3}{3} + C$$

$$f(3) = (3) - \frac{(3)^3}{3} + C = 0$$

$$C = 6$$
$$f(x) = x - \frac{x^3}{3} + 6$$

*Answer:*

.....

**(Total 4 m**

If  $f'(x) = \cos x$ , and  $f\left(\frac{\pi}{2}\right) = -2$ , find  $f(x)$ .

*Working:*

*Answer:*

(Total

83.) A curve with equation  $y = f(x)$  passes through the point  $(1, 1)$ . Its gradient function is  $f'(x) = -2x + 3$ .

Find the equation of the curve.

*Working:*

*Answer:*

(Total

The function  $f$  is such that  $f''(x) = 2x - 2$ .

When the graph of  $f$  is drawn, it has a minimum point at  $(3, -7)$ .

- (a) Show that  $f'(x) = x^2 - 2x - 3$  and hence find  $f(x)$ .
- (b) Find  $f(0)$ ,  $f(-1)$  and  $f'(-1)$ .
- (c) Hence sketch the graph of  $f$ , labelling it with the information obtained in part (b).

**(Note: It is not necessary to find the coordinates of the points where the graph cuts the  $x$ -axis.)**

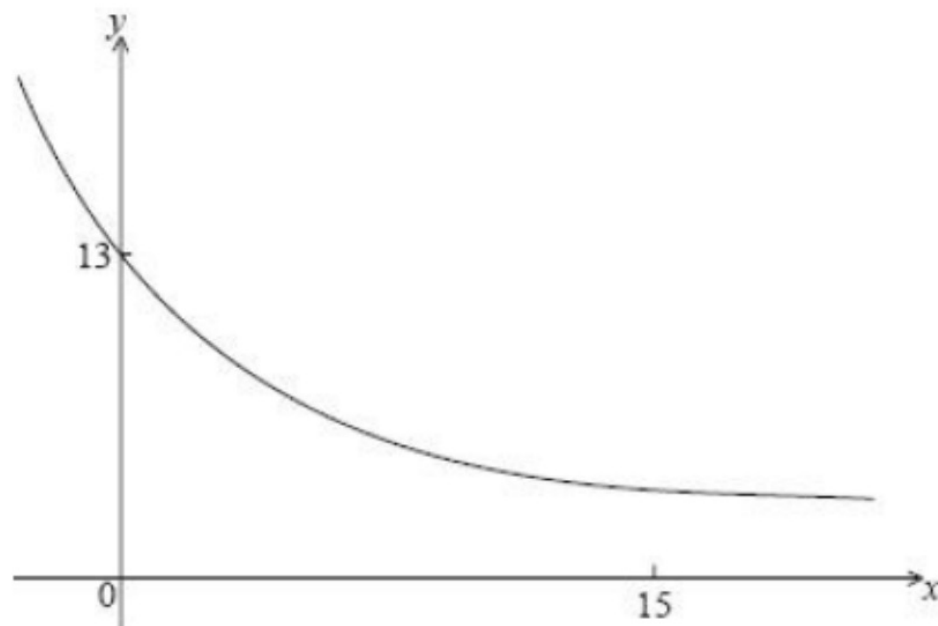
**(Total**

- (i) Find  $\frac{dy}{dx}$  in terms of  $k$ , where  $y = e^{-kx}$ .

The point  $P(1, 0.8)$  lies on the graph of the function  $y = e^{-kx}$ .

- (ii) Find the value of  $k$  in this case.
- (iii) Find the gradient of the tangent to the curve at  $P$ .

Let  $f(x) = Ae^{kx} + 3$ . Part of the graph of  $f$  is shown below.



The  $y$ -intercept is at  $(0, 13)$ .

- (a) Show that  $A = 10$ . (2)
- (b) Given that  $f(15) = 3.49$  (correct to 3 significant figures), find the value of  $k$ . (3)
- (c) (i) Using your value of  $k$ , find  $f'(x)$ .
- (ii) Hence, explain why  $f$  is a decreasing function.
- (iii) Write down the equation of the horizontal asymptote of the graph  $f$ . (5)

(a) Find  $\int \frac{1}{2x+3} dx$ .

(2)

(b) Given that  $\int_0^3 \frac{1}{2x+3} dx = \ln \sqrt{P}$ , find the value of  $P$ .

(4)

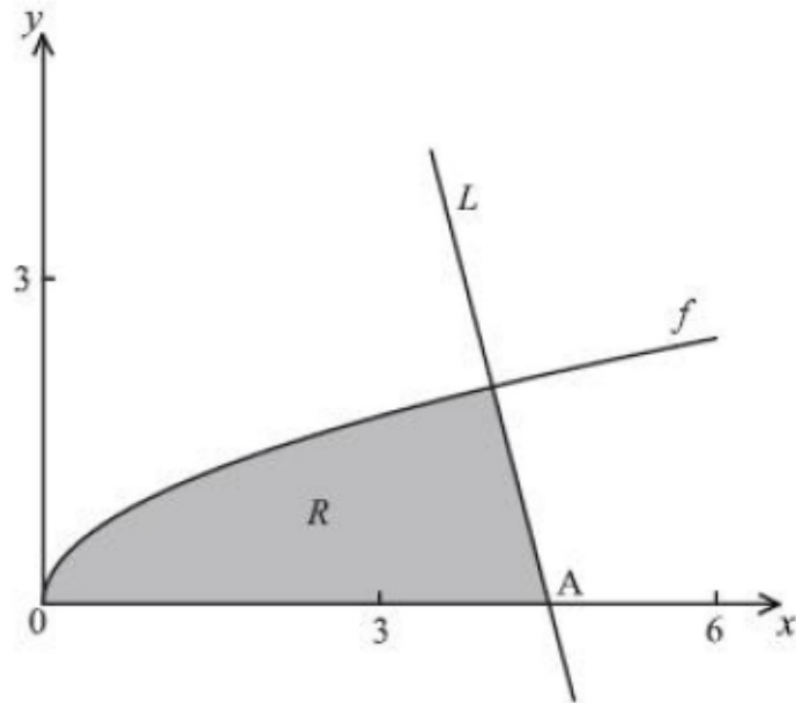
(Total 6 marks)

Let  $f(x) = \sqrt{x}$ . Line  $L$  is the normal to the graph of  $f$  at the point  $(4, 2)$ .

(a) Show that the equation of  $L$  is  $y = -4x + 18$ . (4)

(b) Point  $A$  is the  $x$ -intercept of  $L$ . Find the  $x$ -coordinate of  $A$ . (2)

In the diagram below, the shaded region  $R$  is bounded by the  $x$ -axis, the graph of  $f$  and the line  $L$ .

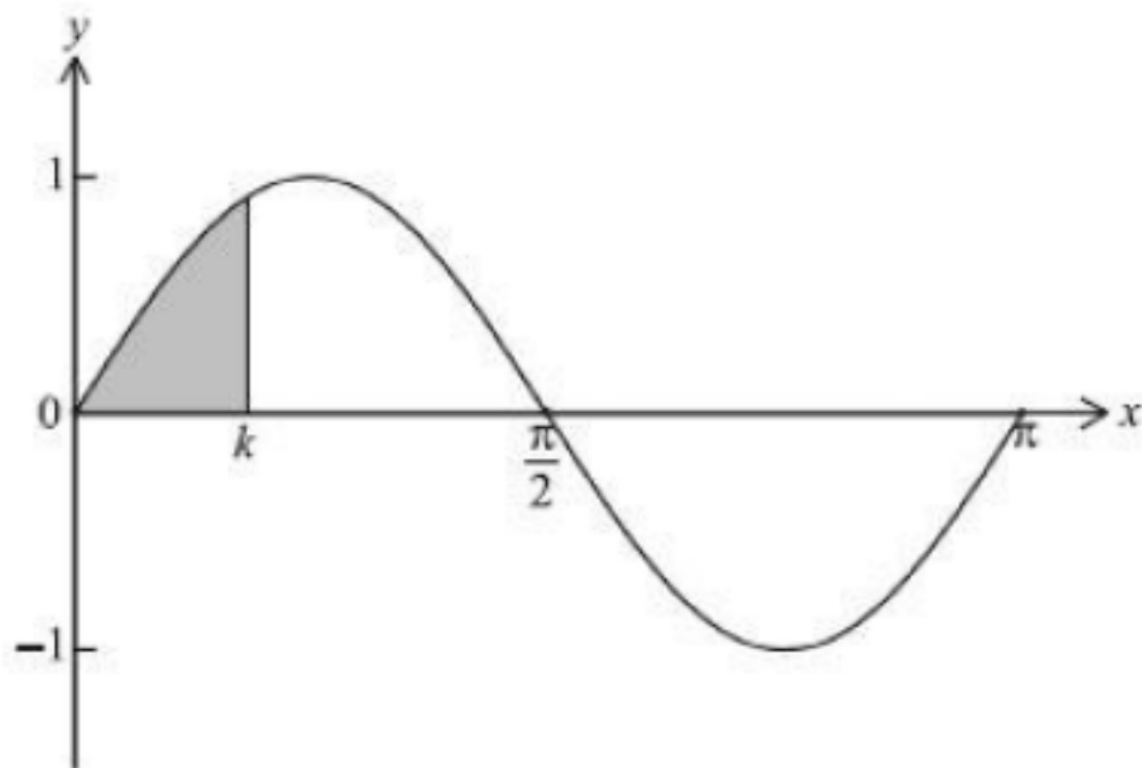


(c) Find an expression for the area of  $R$ .



Given  $\int_3^k \frac{1}{x-2} dx = \ln 7$ , find the value of  $k$ .

The graph of  $y = \sin 2x$  from  $0 \leq x \leq \pi$  is shown below.



The area of the shaded region is  $0.85$ . Find the value of  $k$ .

Let  $f$  be a function such that  $\int_0^3 f(x) dx = 8$ .

(a) Deduce the value of

(i)  $\int_0^3 2f(x) dx$ ;

(ii)  $\int_0^3 (f(x) + 2) dx$ .

(b) If  $\int_c^d f(x-2) dx = 8$ , write down the value of  $c$  and of  $d$ .

Given that  $\int_1^3 g(x)dx = 10$ , deduce the value of

(a)  $\int_1^3 \frac{1}{2} g(x)dx;$

(b)  $\int_1^3 (g(x) + 4)dx.$