Do you like turducken?

Find the inverse for $y = 5x - 2$

$f(x) = 5x - 2$

find $f^{-1}(x)$

$x = 5y - 2$

$y = \frac{x+2}{5} = f^{-1}(x)$

Today's learning objective:

By the end of class, I will be able to write inverse and composite functions.

Today's language objective:

$f^{-1}(x)$: inverse

exchange variables
$x \to y$
$y \to x$

$f(g(x))$
$(f \cdot g)(x)$ } composite

B version

[Maximum mark: 6]

The functions f and g are defined by $f(x) \mapsto 3x$, $g : x \mapsto x + 2$.

(a) Find an expression for $(f \circ g)(x)$. *[2 ma*

(b) Find $f^{-1}(18) + g^{-1}(18)$. *[4 ma*

a) $(f \circ g)(x) = 3(x+2) = 3x+6$

b) $y = 3x$ $\quad y = x+2$

$x = 3y$ $\quad x = y+2$

$18 = 3y$ $\quad 18 = y+2$

$y = 6$ $\quad y = 16$

22

Let $f(x) = k \log_2 x$.

(a) Given that $f^{-1}(1) = 8$, find the value of k. $= 1/3$ [3 m

(b) Find $f^{-1}\left(\frac{2}{3}\right)$. [4 m

$k \cdot \frac{1}{k3} = \frac{3 \cdot k}{3}$

$y = k \log_2 x$

$\frac{x}{k} = \frac{k}{k} \log_2 y$

$\frac{x}{k} = \log_2 y$

$1 = x/k$

$2 = y = 8$

Consider the functions f and g where $f(x) = 2x - 3$ and $g(x) = x - 4$.

(a) Find the inverse function, f^{-1} and g^{-1} **(4)**

$f^{-1}(x) = \frac{x+3}{2}$ $\quad g^{-1}(x) = x + 4$

(b) Find $(g^{-1} \circ f)(x)$. **(2)**

$(2x-3)+4 = 2x+1$

(c) Given that $(f^{-1} \circ g)(x) = \frac{x-1}{2}$, solve $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$. **(2)**

$\frac{x-1}{2} = 2x+1$ $\quad x = -1$

Let $h(x) = \frac{f(x)}{g(x)}$, $x \neq 2$. $= \frac{2x-3}{x-4}$ $\quad y = 2$ asymptote, $x = 4$ asymptote

(d) (i) **Sketch** the graph of h for $-3 \leq x \leq 8$ and $-2 \leq y \leq 8$, including any asymptotes.

(ii) Write down the **equations** of the asymptotes. **(5)**

$\int \frac{1}{x} dx = \ln x + c$

(e) The expression $\frac{2x-3}{x-4}$ may also be written as $2 + \frac{5}{x-4}$. Use this to answer the following.

(i) Find $\int h(x)\,dx$. $= 2x + 5\ln(x-4) + c$

(ii) **Hence**, calculate the **exact** value of $\int_{4.5}^{7} h(x)\,dx$. **(5)**

$(14 + 5\ln 3) - (9 + 5\ln 0.5)$

(f) On your sketch, shade the region whose area is represented by $\int_{4.5}^{7} h(x)\,dx$.

$$5 + 5 \ln 3 - 5 \ln 0.5$$

$$5 + 5(\ln 3 - \ln 0.5)$$

$$\boxed{5 + 5 \ln 6} \approx 14.0$$

[Maximum marks 7]

Let $f(x) = \ln(x + 2) + \ln 5$, for $x > -5$.

(a) Find $f^{-1}(x)$. (4)

Let $g(x) = e^x + 2$

(b) Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b \in \mathbb{Z}$. (3)

$e^x = 5y + 10$

$\ln(5x+10) = y$

$\ln(5y+10) = x$

$\log_e(5y+10) = x$

$\frac{e^x - 10}{5} = \frac{5y}{5}$

$e^{\ln(5x+10)} + 2 = y - 2$

$\log_e(y-2) = \ln(5x+10)$

$\log_e(y-2) = \log_e(5x+10)$

1.) Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$. $= 10 - 2x$

$(7 - 2x) + 3$

(b) Write down $g^{-1}(x)$. $= x - 3$

(c) Find $(f \circ g^{-1})(5)$.

$7 - 2(x - 3)$

$7 - 2x + 6$

$13 - 2x$

3

(Total 5 r

9.) Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

non-calc

$3^? = 9$

(a) Show that $f(x) = \log_3 2x$.

(2)

(b) Find the value of $f(0.5)$ and of $f(4.5)$.

0 2

$\log_3 2(0.5) = \log_3 1 = ?$

(3)

The function f can also be written in the form $f(x) = \frac{\ln dx}{\ln b}$ $= \log_3 2x$ $\frac{\log_e 2x}{\log_e 3}$

(c) (i) Write down the value of a and of b.

2 3

(ii) Hence on graph paper, **sketch** the graph of f, for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

(6)

(d) Write down the value of $f^{-1}(0)$.

(1)

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A.

(4)

(Total 16 marks)

10.) Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$. $= 6x - 15$

(b) Find $h^{-1}(x)$. $= \dfrac{x + 15}{6}$

(Total 5 r

13.) Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$.

(2)

(b) Write down the range of f^{-1}.

(1)

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

(4)

(Total 7 marks)

16.) Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

(a) Find $f\left(\frac{\pi}{2}\right)$.

(2)

(b) Find $(g \circ f)\left(\frac{\pi}{2}\right)$.

(2)

(c) Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of $k, k \in \mathbb{Z}$.

(3)

(Total 7 marks)

25.) Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$.

(2)

The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h.

(b) Find the coordinates of the vertex of the graph of h.

(3)

(c) Show that $h(x) = x^2 - 8x + 19$.

(2)

(d) The line $y = 2x - 6$ is a tangent to the graph of h at the point P. Find the x-coordinate of P.

(5)

(Total 12 marks)