Today's learning objective:

By the end of class, I will be able to solve rational and quadratic function problems and interpret curved lines.

$3^{2\log_3 2} = ?$

$\log_3 ? = 2\log_3 2$

$\log_3 ? = \log_3 2^2$

Today's language objective:

Horizontal asymptote
Vertical asymptote
p(x) and q(x)
Vertex form vs Standard form
Minimum vs Maximum
Axis of Symmetry

extrane $\frac{0}{0}$us

89.) Let $g(x) = 3x - 2$, $h(x) = \dfrac{5x}{x-4}$, $x \neq 4$.

(a) Find an expression for $(h \circ g)(x)$. Simplify your answer.

(b) Solve the equation $(h \circ g)(x) = 0$.

(Total 6 m

$$\frac{5(3x-2)}{(3x-2)-4} = \frac{15x-10}{3x-6} = 0$$

$$\frac{0}{15} = 0$$

$$3\left(\frac{2}{3}\right) - 6$$

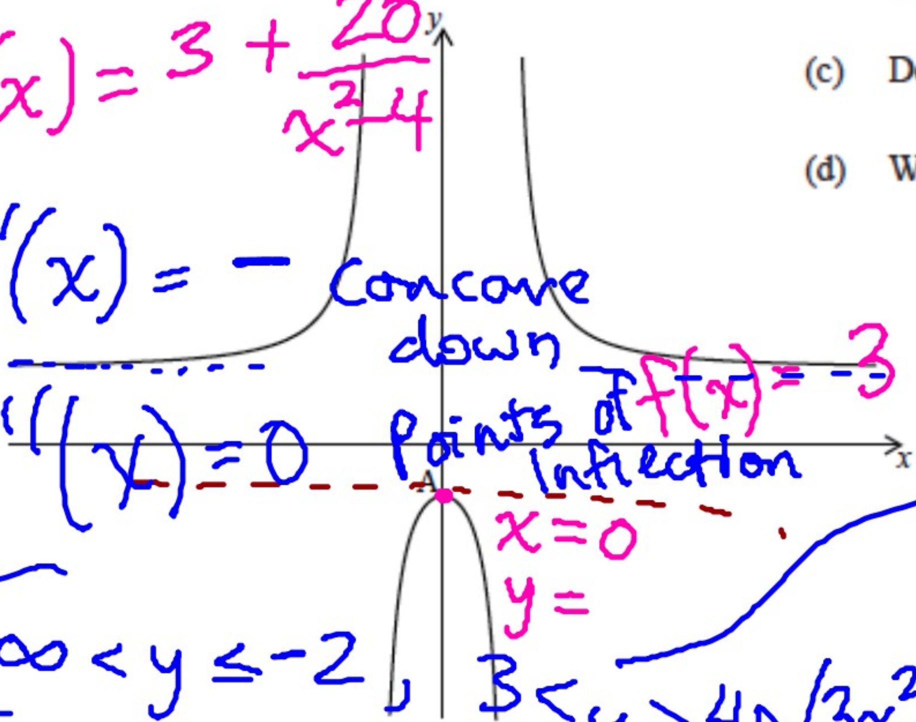
$$15x - 10 = 0$$

$$\frac{15}{0} = \infty$$

$$x = \frac{2}{3}$$

Let $f(x)=3+\dfrac{20}{x^2-4}$, for $x \neq \pm 2$. The graph of f is given below.

$f(x) = 3 + \dfrac{20}{x^2-4}$



$f''(x) = -$ concave down

$f''(x) = 0$ Points of inflection

$f(x) = -3$

$x = 0$

$y =$

$-\infty < y \leq -2, \; 3 < y < \infty$

The y-intercept is at the point A.

(a) (i) Find the coordinates of A. $(0, -2)$

(ii) Show that $f'(x) = 0$ at A. *[7 marks]*

$20(x^2-4)^{-1}$

$f'(x) = -20 \cdot 2x(x^2-4)^{-2}$

$f'(x) = \dfrac{-40x}{(x^2-4)^2}$

(b) The second derivative $f''(x) = \dfrac{40(3x^2+4)}{(x^2-4)^3} = 0$. Use this to

$40(3x^2+4) \neq 0$

$\dfrac{160}{-64}$

(i) justify that the graph of f has a local maximum at A;

(ii) explain why the graph of f does not have a point of inflexion. *[6 marks]*

(ii) explain why the graph of f does not have a point of i

(c) Describe the behaviour of the graph of f for large $|x|$.

As $|x| \to \infty$, $f(x) \to 3$

(d) Write down the range of f.

72.) The function $f(x)$ is defined as $f(x) = 3 + \dfrac{1}{2x-5}$, $x \neq \dfrac{5}{2}$.

(a) Sketch the curve of f for $-5 \leq x \leq 5$, showing the asymptotes.

(3)

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the x-intercept;

(iii) the value of the y-intercept.

(4)

(c) The region enclosed by the curve of f, the x-axis, and the lines $x = 3$ and $x = a$, is revolved through $360°$ about the x-axis. Let V be the volume of the solid formed.

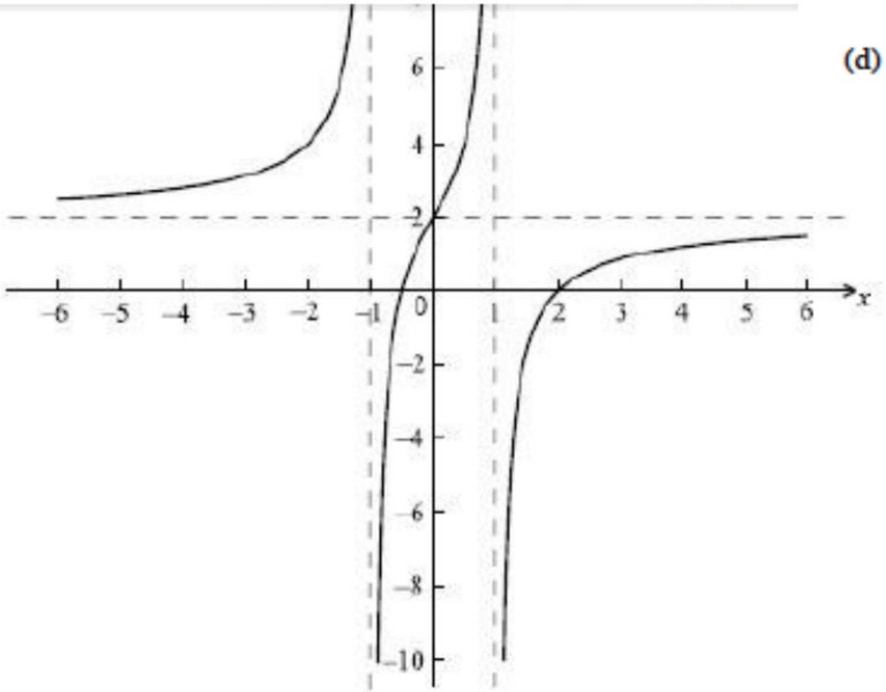
(i) Find $\displaystyle\int\left(9+\frac{6}{2x-5}+\frac{1}{(2x-5)^2}\right)\mathrm{d}x$.

(ii) Hence, given that $V = \pi\left(\dfrac{28}{3}+3\ln 3\right)$, find the value of a.

(10)
(Total 17 marks)

73.) Let $f(x) = p - \frac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$.

Part of the graph of f, including the asymptot



(a) The equations of the asymptotes are $x=1$, $x=-1$, $y=2$. Write down the value of

(i) p;

(ii) q.

(2)

(b) Let R be the region bounded by the graph of f, the x-axis, and the y-axis.

(i) Find the negative x-intercept of f.

(ii) Hence find the volume obtained when R is revolved through 360° about the x-axis.

(c) (i) Show that $f'(x) = \frac{3(x^2+1)}{(x^2-1)^2}$.

(ii) Hence, show that there are no maximum or minimum points or

(d) Let $g(x) = f'(x)$. Let A be the area of the region enclosed by the grap between $x=0$ and $x=a$, where $a>0$. Given that $A=2$, find the value

ax^2+bx+c

2.) Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k. $= \frac{1}{2}$

$x^3(x-2)^4$

non-calc

(b) The line $y = p$ intersects the graph of f. Find all possible values of p.

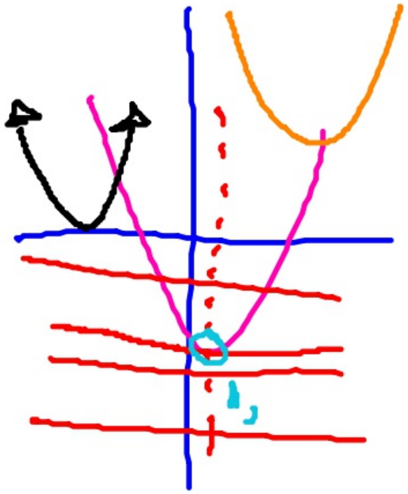
(To

discriminant $= b^2 - 4ac$

$= +$, 2 roots

$= -$, 0 roots

$= 0$, 1 root



$(-4k)^2 - 4(2k)(1)$

$16k^2 - 8k = 0$

$8k(2k-1) = 0$

$2k - 1 = 0$

$+1 \quad +1$

$2k = 1$

$k = 1/2$

$f(x) = 1x^2 - 2x + 1$

$(x-1)(x-1)$

$p \geq 0$

Axis of symmetry

$x = \frac{-b}{2a} = \frac{2}{2} = 1$

Let $\overset{y}{\cancel{f(x)}} = ax^2 + bx + c$ where a, b and c are rational numbers. fractional

(a) The point P(–4, 3) lies on the curve of f. Show that

x y

(b) The points Q(6, 3) and R(–2, –1) also lie on the curve of f. Write down two other linear equations in a, b and c.

$$3 = a(-4)^2 + b(-4) + c$$

$$3 = 16a - 4b + c$$

Axis of Symmetry

$$x = \frac{-b}{2a}$$

) The quadratic equation $kx^2 + (k-3)x + 1 = 0$ has two equal real roots.

(a) Find the possible values of k.

(b) **Write down** the values of k for which $x^2 + (k-3)x + k = 0$ has two equal real roots.

(Total 7

$y = ax^2 + bx + c$

A.o.S.

$x = \frac{-b}{2a}$

Discriminant

$b^2 - 4ac = 0$

$(k-3)^2 - 4(k)(1) = 0$

$k^2 - 6k + 9 - 4k = 0$

$k^2 - 10k + 9 = 0$

$(k \quad -9)$

$(k \quad -1)$

$\boxed{\begin{matrix} k = 9 \\ k = 1 \end{matrix}}$

52.) Let $f(x) = \underbrace{2x^2 + 4x - 6}$.

(a) Express $f(x)$ in the form $f(x) = \underbrace{2(x-h)^2 + k}_{\text{vertex}}$. $\boxed{2(x+1)^2 - 8}$

Vertex $= (^-1, ^-8)$

(b) Write down the equation of the axis of symmetry of the graph of f.

$x = -1$

(c) Express $f(x)$ in the form $f(x) = 2(x-p)(x-q)$.

$2(x^2 + 2x - 3)$

$x \quad +3$

$x \quad -1$

non-calc

(Total 6

A.o.S.

$x = \frac{^-b}{2a}$

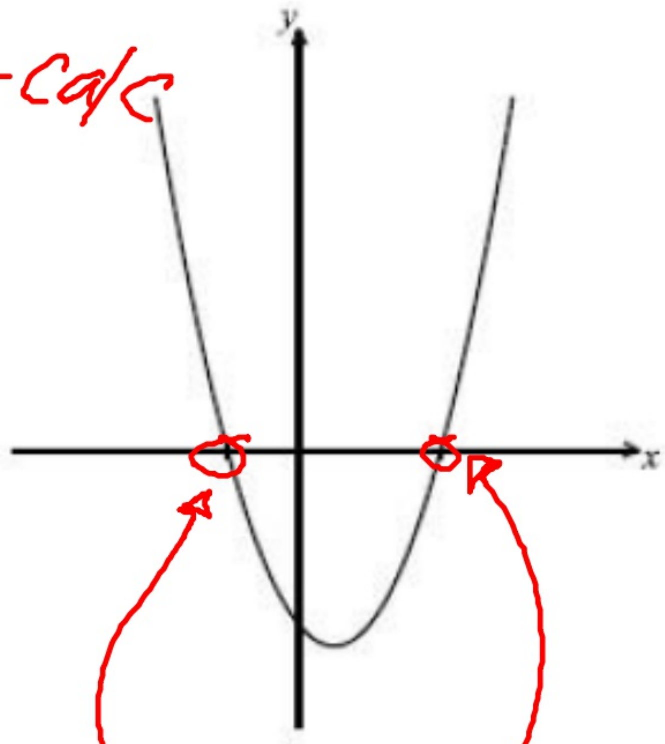
$2(x+3)(x-1)$

$x = \frac{-4}{2(2)} = ^-1$

45.) The following diagram shows part of the graph of f, where $f(x) = x^2 - x - 2$.

$(x-2)(x+1)$

non-calc



(a) Find both x-intercepts. $x = 2 \quad x = -1$

(b) Find the x-coordinate of the vertex.

A.O.S

$x = \frac{-b}{2a}$

$x = \frac{1}{2}$

(Total 6 r

58.) Let $f(x) = 2x^2 - 12x + 5$.

(a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 - k$.

(3)

(b) Write down the vertex of the graph of f.

(2)

(c) Write down the equation of the axis of symmetry of the graph of f.

(1)

(d) Find the y-intercept of the graph of f.

(2)

(e) The x-intercepts of f can be written as $\frac{p \pm \sqrt{q}}{r}$, where $p, q, r \in \mathbb{Z}$.

Find the value of p, of q, and of r.

(7)

(Total 15 marks)