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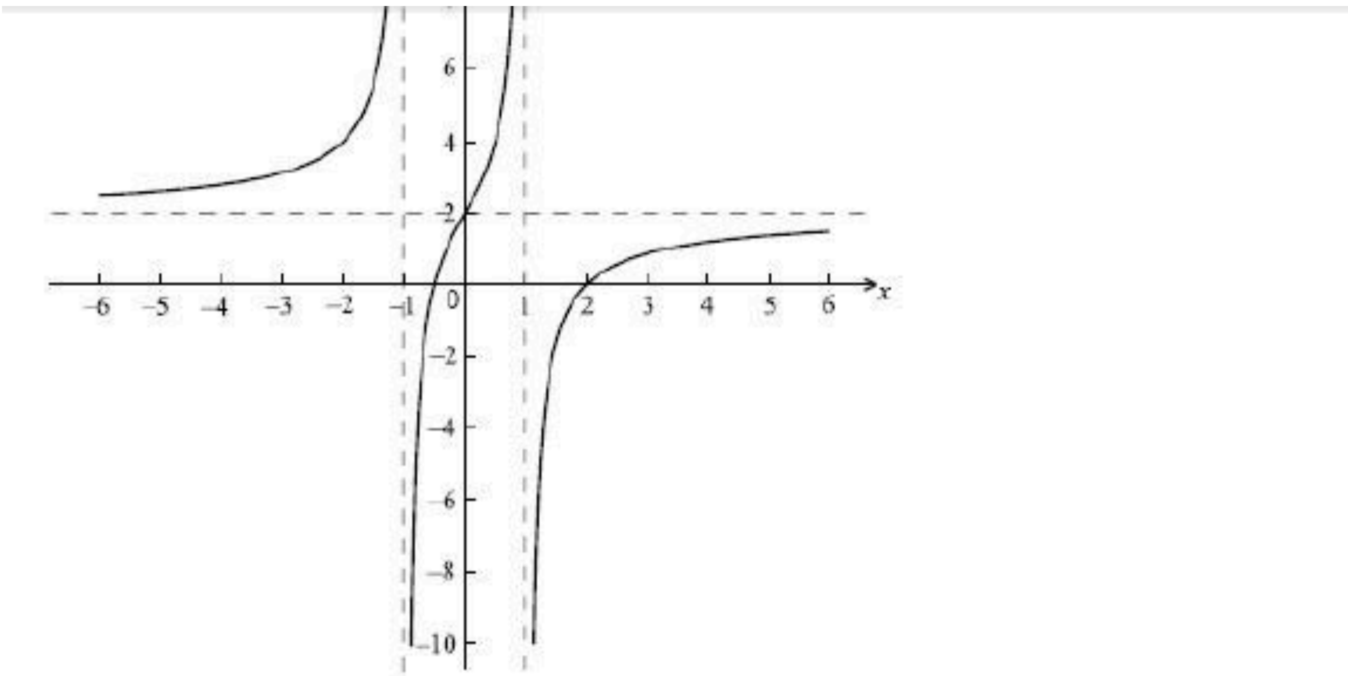
## <u>ASSIGNMENT</u>: Rational Functions *[SL calc]*

73.) Let $f(x) = p - \dfrac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$.

Part of the graph of $f$, including the asymptotes, is shown below.




(a) The equations of the asymptotes are $x = 1$, $x = -1$, $y = 2$. Write down the value of

(i) $p$;

(ii) $q$.

**(2)**

(b) Let $R$ be the region bounded by the graph of $f$, the $x$-axis, and the $y$-axis.

(i) Find the negative $x$-intercept of $f$.

(ii) Hence find the volume obtained when $R$ is revolved through 360° about the $x$-axis.

**(7)**

(c) (i) Show that $f'(x) = \dfrac{3(x^2+1)}{(x^2-1)^2}$.

(ii) Hence, show that there are no maximum or minimum points on the graph of $f$.

**(8)**

(d) Let $g(x) = f'(x)$. Let $A$ be the area of the region enclosed by the graph of $g$ and the $x$-axis, between $x = 0$ and $x = a$, where $a > 0$. Given that $A = 2$, find the value of $a$.

**(7)**

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(a) (i) $p = 2$ A1 N1

(ii) $q = 1$ A1 N1

(b) (i) $f(x) = 0$ (M1)

$2 - \frac{3x}{x^2 - 1} = 0 \quad (2x^2 - 3x - 2 = 0)$ A1

$x = -\frac{1}{2}\ x = 2$

$\left(-\frac{1}{2}, 0\right)$ A1 N2

(ii) Using $V = \int_a^b \pi y^2 \mathrm{d}x$ (limits not required) (M1)

$V = \int_{\frac{1}{2}}^{0} \pi \left(2 - \frac{3x}{x^2 - 1}\right)^2 \mathrm{d}x$ A2

$V = 2.52$ A1 N2

(c) (i) Evidence of appropriate method M1

*eg* Product or quotient rule

Correct derivatives of $3x$ and $x^2 - 1$ A1A1

Correct substitution A1

*eg* $\frac{-3(x^2 - 1) - (-3x)(2x)}{(x^2 - 1)^2}$

$f'(x) = \frac{-3x^2 + 3 + 6x^2}{(x^2 - 1)^2}$ A1

$f'(x) = \frac{3x^2 + 3}{(x^2 - 1)^2} = \frac{3(x^2 + 1)}{(x^2 - 1)^2}$ AG N0

(ii) **METHOD 1**

Evidence of using $f'(x) = 0$ at max/min (M1)

$3(x^2 + 1) = 0\ (3x^2 + 3 = 0)$ A1

no (real) solution R1

Therefore, no maximum or minimum. AG N0

**METHOD 2**

Evidence of using $f'(x) = 0$ at max/min (M1)

Sketch of $f'(x)$ with good asymptotic behaviour A1

Never crosses the $x$-axis R1

Therefore, no maximum or minimum. AG N0

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**METHOD 3**

| | |
|---|---|
| Evidence of using $f'(x) = 0$ at max/min | (M1) |
| Evidence of considering the sign of $f'(x)$ | A1 |

| | | |
|---|---|---|
| $f'(x)$ is an increasing function ($f'(x) > 0$, always) | R1 | |
| Therefore, no maximum or minimum. | AG | N0 |

(d) For using integral (M1)

Area $= \int_0^a g(x)\,dx \left(\text{or } \int_0^a f'(x)\,dx \text{ or } \int_0^a \frac{3x^2+3}{(x^2-1)^2}\,dx\right)$ A1

Recognizing that $\int_0^a g(x)\,dx = f(x)\Big|_0^a$ A2

Setting up equation (seen anywhere) (M1)

Correct equation A1

*eg* $\int_0^a \frac{3x^2+3}{(x^2-1)^2}\,dx = 2$, $\left[2-\frac{3a}{a^2-1}\right]-[2-0]=2$, $2a^2+3a-2=0$

$a = \frac{1}{2}$ $\quad a = -2$