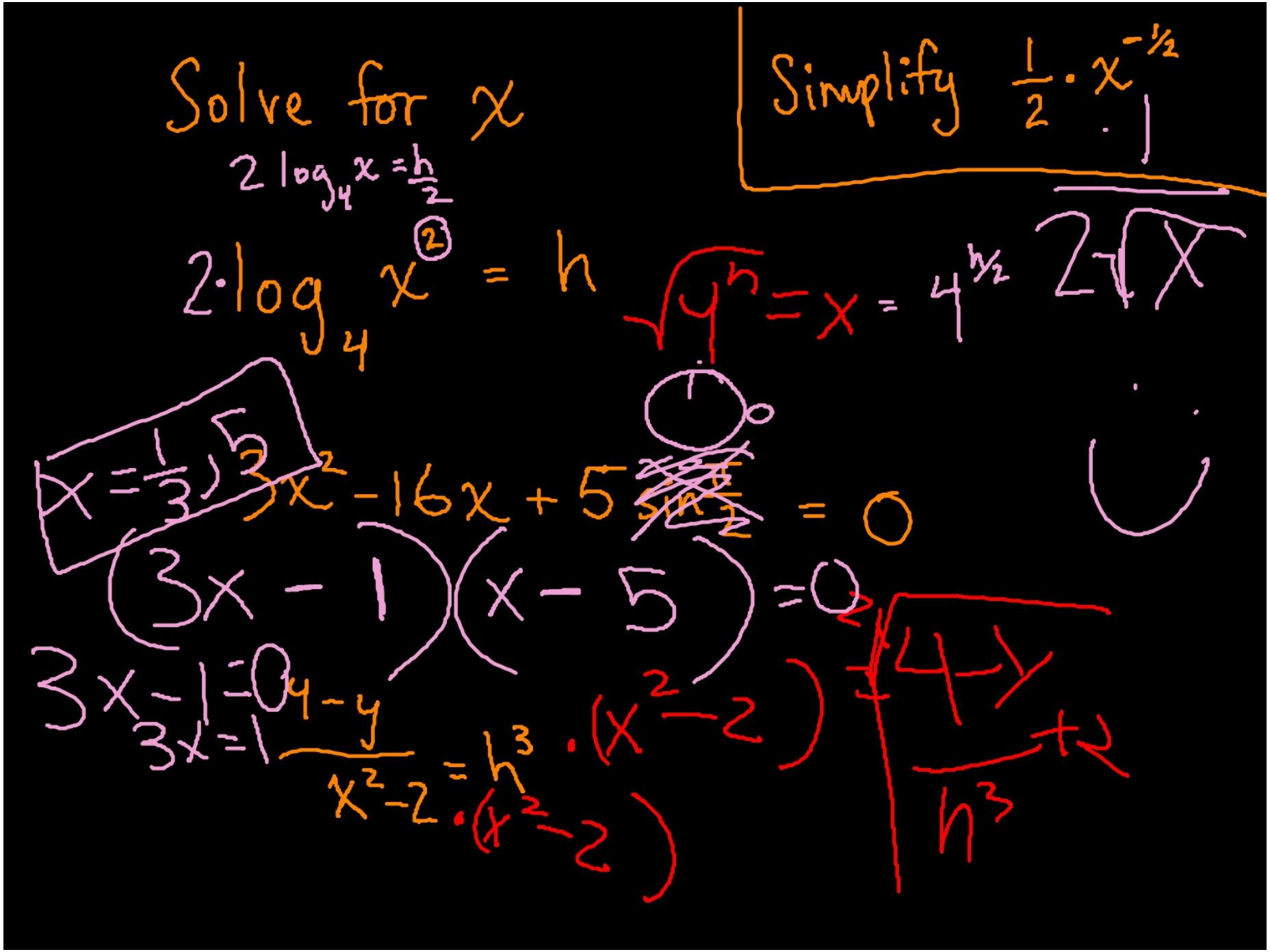
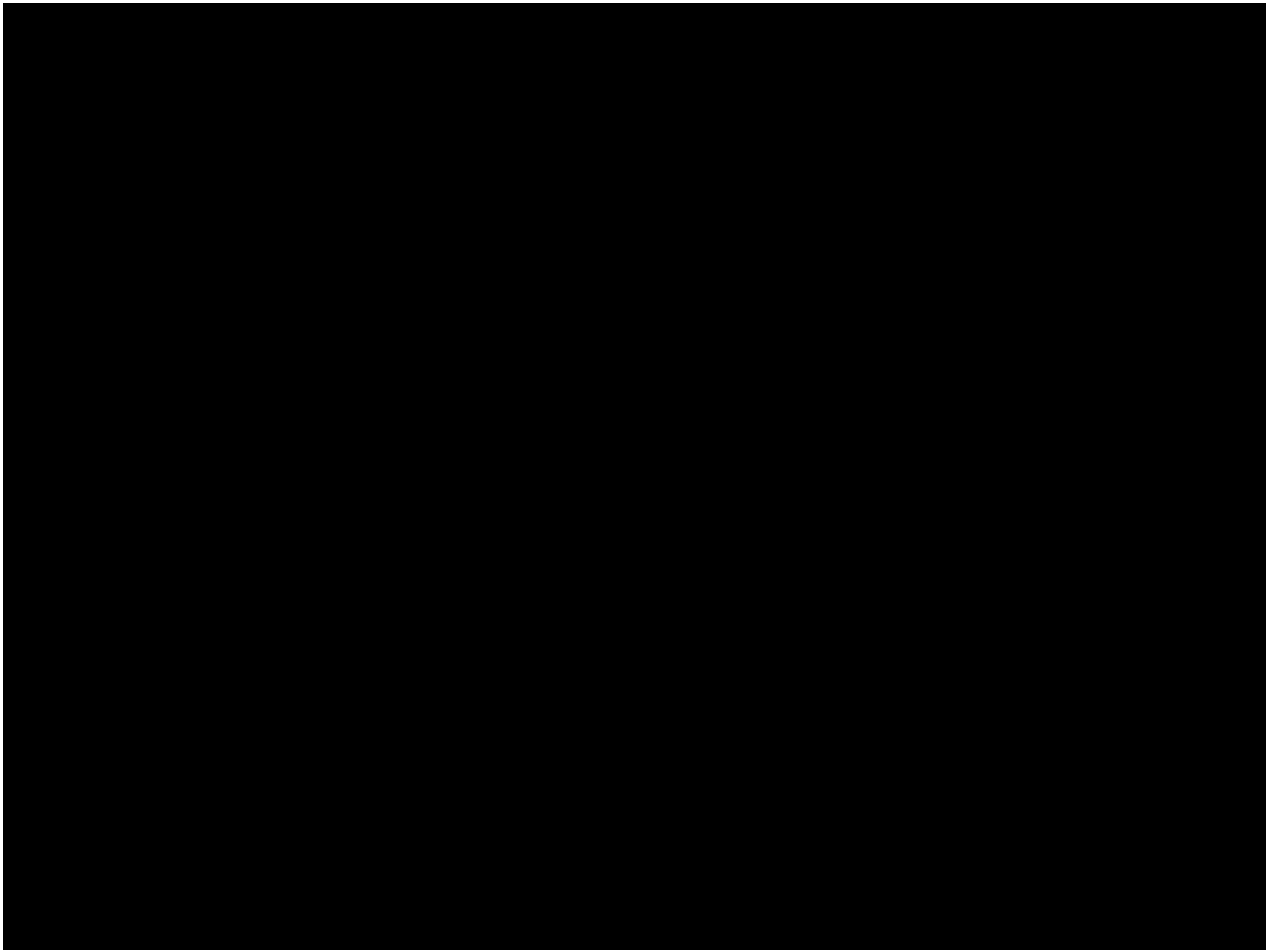

Solve for x
$2\log_4 x = \frac{h}{2}$
$2 \cdot \log_4 x^{②} = h$
$\sqrt{4^h} = x = 4^{h/2}$
$x = \frac{1}{3}, 5$
$3x^2 - 16x + 5 = 0$
$(3x - 1)(x - 5) = 0$
$3x - 1 = 0$
$3x = 1$
$\frac{4-y}{x^2-2} = \frac{h^3}{x^2-2} \cdot (x^2-2)$
$\frac{4-y}{h^3} + 2$
Simplify $\frac{1}{2} \cdot x^{-1/2}$
$2\sqrt{x}$



What units could you expect in an area problem?

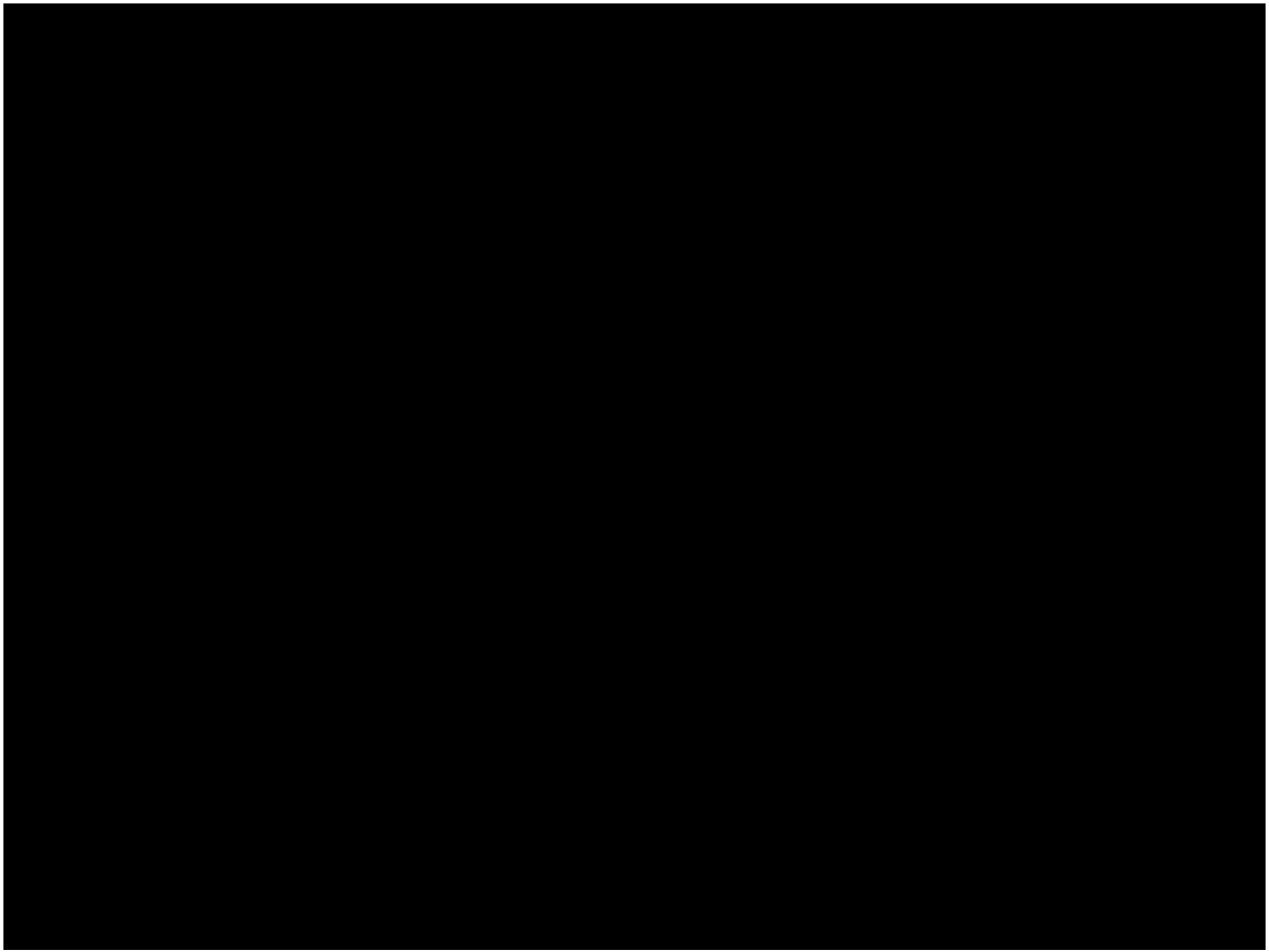
$\text{units}^2$ $\int [f(x)]^2$

A volume problem?

$\text{units}^3$

$\pi \int_a^b [f(x)]^2 dx$

$\pi \int r^2$



# Today's learning objective:

$2x^2$ $2 \cdot x^2$

$4x = 2 \cdot 2x$

By the end of class, I will be able to calculate volumes of curves rotated around the x-axis.

| | |
|---|---|
| Volume of revolution about the $x$-axis from $x = a$ to $x = b$ | $V = \int_a^b \pi y^2 \, dx$ |

2nd 1st

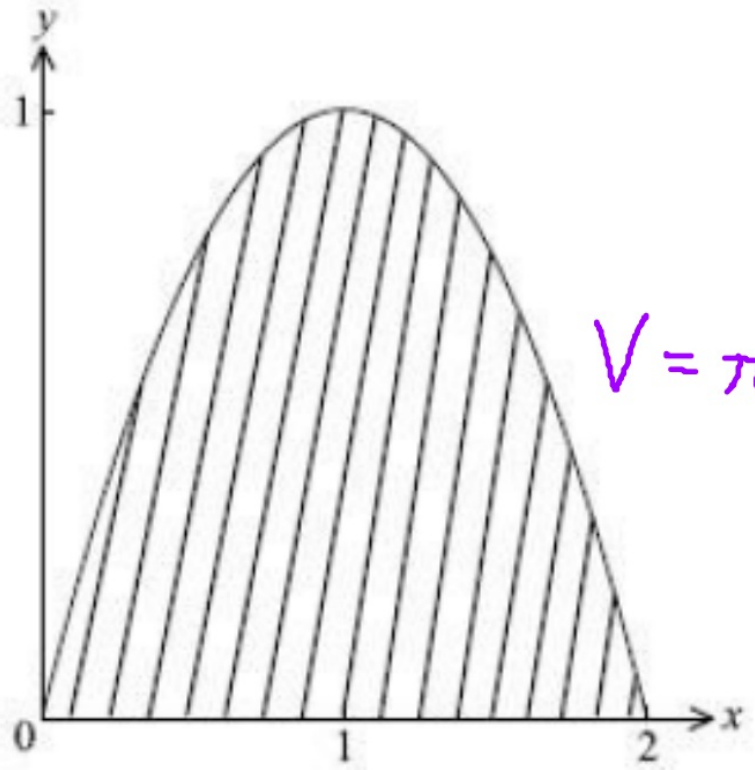
$V = \pi \int_a^b y^2 dx$

last

A part of the graph of $y = 2x - x^2$ is given in the diagram below.

non-calc

$$V = \pi \int_a^b y^2 \, dx$$




$$V = \pi \int_0^2 (2x - x^2)^2 \, dx$$

$$\pi \int_0^2 (4x^2 - 4x^3 + x^4) \, dx$$

$$\pi \left| \frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right|_0^2$$

$$\frac{16\pi}{15}$$

The shaded region is revolved through 360° about the $x$-axis.

(a) Write down an expression for this volume of revolution.

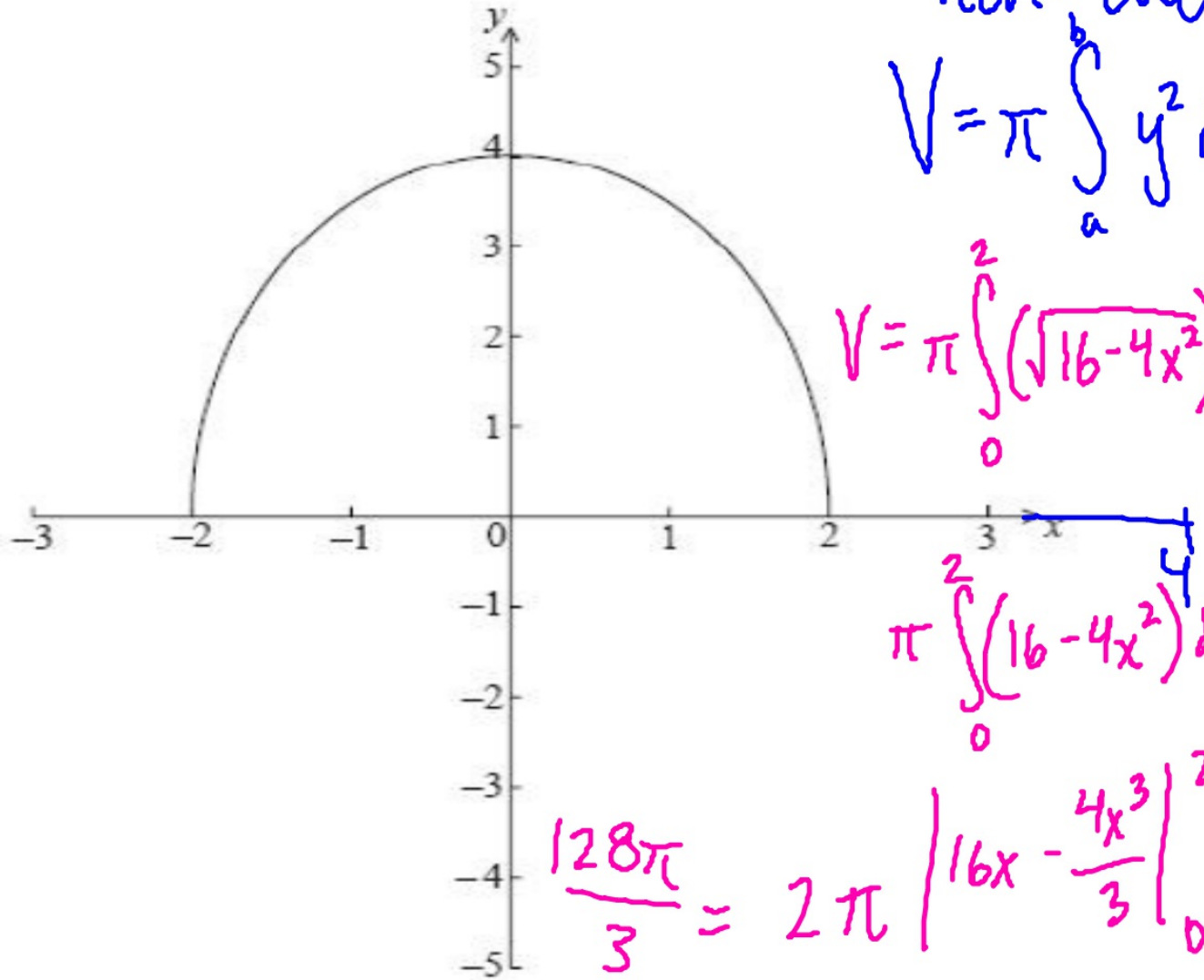
$$\left( \frac{32}{3} - 16 + \frac{32}{5} \right)$$

(b) Calculate this volume.

$$\frac{160}{15} - \frac{240}{15} + \frac{96}{15}$$

(Tota

**The graph of $f(x) = \sqrt{16-4x^2}$, for $-2 \le x \le 2$, is shown below.**




non-calc

$$V = \pi \int_a^b y^2\,dx$$

$$V = \pi \int_0^2 \left(\sqrt{16-4x^2}\right)^2 dx$$

$$\pi \int_0^2 \left(16-4x^2\right) dx$$

$$\frac{128\pi}{3} = 2\pi \left| 16x - \frac{4x^3}{3} \right|_0^2$$

The region enclosed by the curve of $f$ and the $x$-axis is rotated 360° about the $x$-axis. Find the volume of the solid formed.

(Total 6

**15.)** The graph of $y = \sqrt{x}$ between $x = 0$ and $x = a$ is rotated 360° about the $x$-axis. The volume of the solid formed is $32\pi$. Find the value of $a$.

Non-calc

$$32\pi = \pi \int_0^a (\sqrt{x})^2 \, dx$$

$$32\pi = \pi \int_0^a x \, dx$$

$$32\pi = \pi \left| \frac{x^2}{2} \right|_0^a$$

$$32\pi = \frac{\pi a^2}{2}$$

$$a = 8$$

Let $f(x) = \sqrt{x}$ . Line $L$ is the normal to the graph of $f$ at the point (4, 2).

non-calc

(a) Show that the equation of $L$ is $y = -4x + 18$.

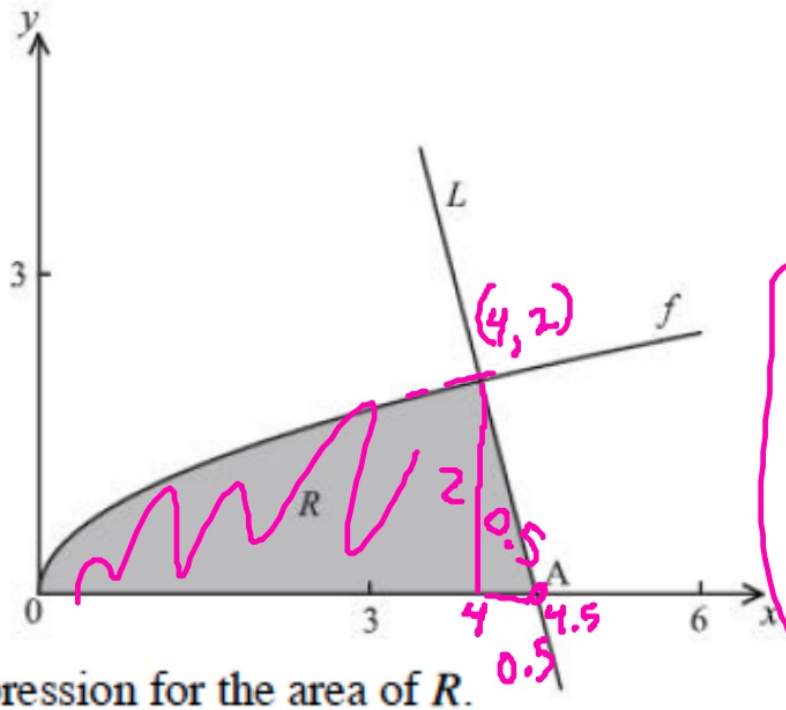
**(4)**

(b) Point A is the $x$-intercept of $L$. Find the $x$-coordinate of A.

$0 = -4x + 18 \quad x = 4.5$

**(2)**

In the diagram below, the shaded region $R$ is bounded by the $x$-axis, the graph of $f$ and the line $L$.




$A = \frac{bh}{2}$

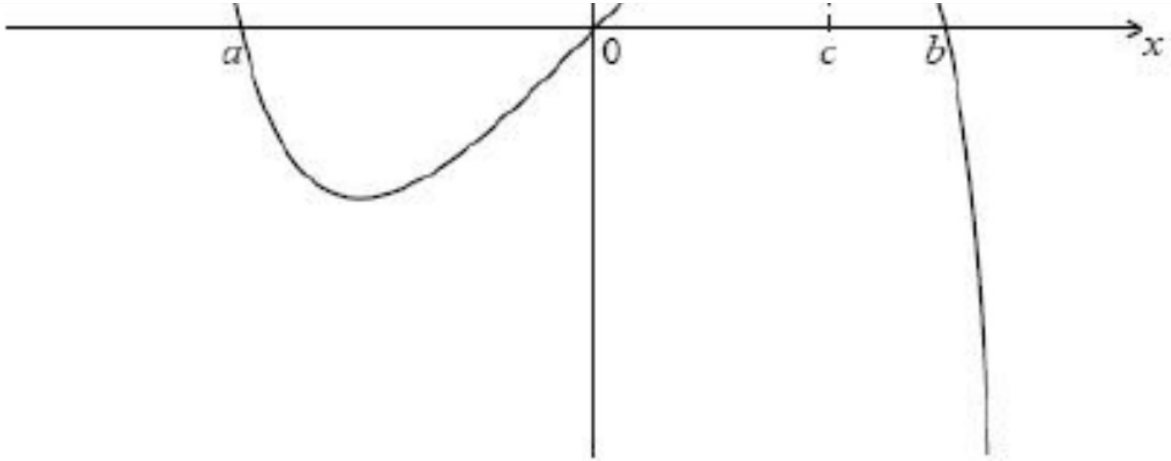
$\int_0^4 \sqrt{x}\,dx + \frac{1}{2}$

(c) Find an expression for the area of $R$.

$\frac{1}{3}\pi r^2 h$

(d) The region $R$ is rotated 360° about the $x$-axis. Find the volume of the solid formed, giving your answer in terms of $\pi$.

$8\pi + \Delta$




The graph of $f$ crosses the $x$-axis at $x = a$, $x = 0$ and $x = b$.

(a) Find the value of $a$ and of $b$.

**(3)**

The graph of $f$ has a maximum value when $x = c$.

(b) Find the value of $c$.

**(2)**

(c) The region under the graph of $f$ from $x = 0$ to $x = c$ is rotated 360° about the $x$-axis. Find the volume of the solid formed.

**(3)**

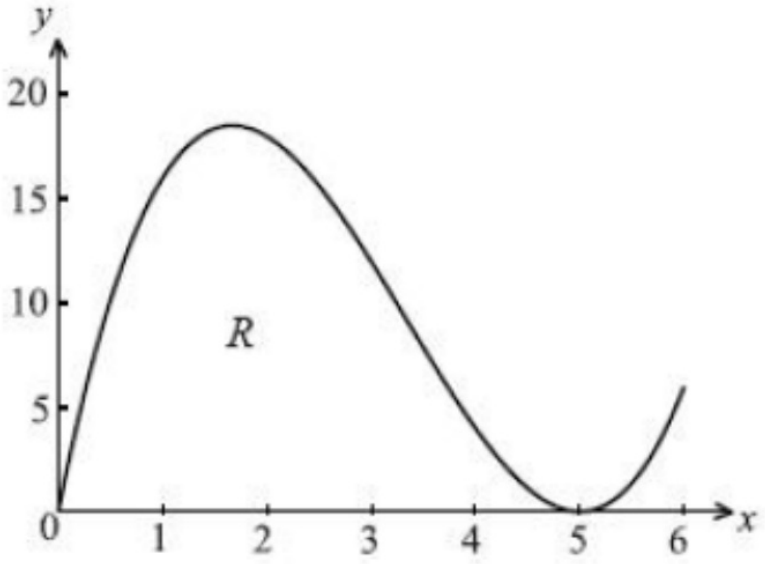
(d) Let $R$ be the region enclosed by the curve, the $x$-axis and the line $x = c$, between $x = a$ and $x = c$.

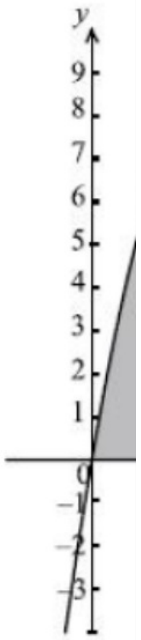
Find the area of $R$.

**(4)**

**(Total 12 marks)**

Let $f(x) = x(x-5)^2$, for $0 \leq x \leq 6$. The following diagram shows the graph of $f$.




Let $R$ be the region enclosed by the $x$-axis and the curve of $f$.

The area of the shaded region is e

(a) Find the area of $R$.

**(3)**

(b) Find the volume of the solid formed when $R$ is rotated through 360° about the $x$-axis.

**(4)**

(c) The diagram below shows a part of the graph of a quadratic function $g(x) = x(a-x)$. The graph of $g$ crosses the $x$-axis when $x = a$.

Let $f : x \mapsto \sin^3 x$.

(a) (i) Write down the range of the function $f$.

(ii) Consider $f(x) = 1, 0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

**(5)**

(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$.

**(2)**

(c) Let $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of $g$ is revolved through $2\pi$ about the $x$-axis.

**(7)**

**(Total 14 marks)**

Calc

14, 18, 19, 28, 42, 52

Non Calc

5, 15, 43, 44, 47, 90