

Why is it called "integration?"

## Standard integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int e^x dx = e^x + C$$

Area under a curve  
between  $x = a$  and  $x = b$

$$A = \int_a^b y dx$$

Volume of revolution  
about the  $x$ -axis from  $x = a$   
to  $x = b$

$$V = \int_a^b \pi y^2 dx$$

## Today's learning objective:

By the end of class, I will be able to integrate using the reverse chain rule.

Find  
 $f'(x)$

$$(3x^2 + 4)^6$$

## Today's language objective:

*differential calc still!*

## Reverse Chain Rule

$$\begin{aligned} &6(3x^2 + 4)^5 \\ &36x(3x^2 + 4)^5 \end{aligned}$$

$$\int \frac{(3x^2 + 4)^7}{7 \cdot 6x} = \frac{(3x^2 + 4)^7}{42x}$$

$$\frac{42x(3x^2 + 4)^6}{42x} = \frac{7(3x^2 + 4)^6}{42x}$$

[Maximum mark: 7]

(a) Find  $\int_1^2 (3x^2 - 2) dx$ .

$$\frac{\cancel{3}x^3}{\cancel{3}} - 2x$$

non-calc

$$x^3 - 2x$$

$$\left[ (2)^3 - 2(2) \right]$$

$$- \left[ (1)^3 - 2(1) \right]$$

$$8 - 4 - 1 + 2$$

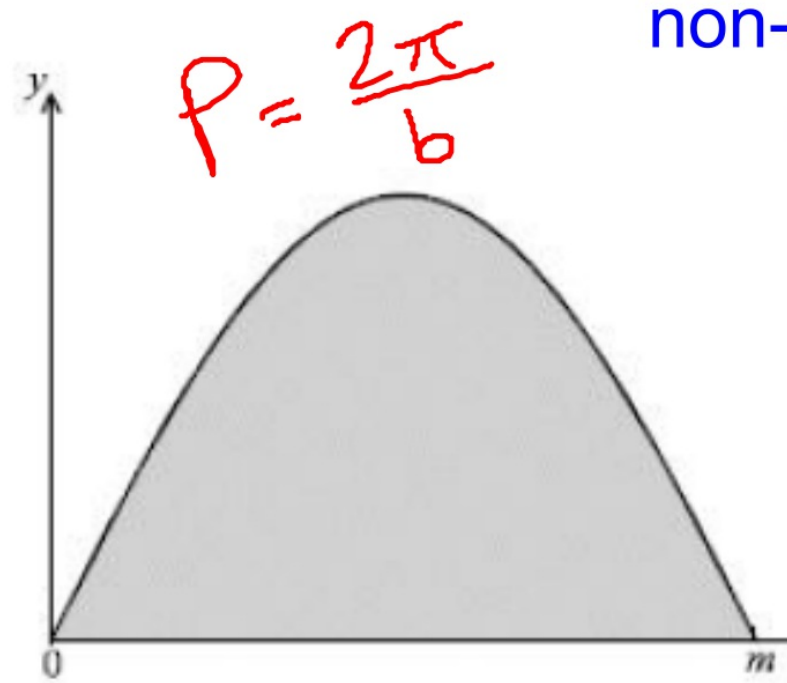
(b) Find  $\int_0^1 2e^{2x} dx$ .

$$\frac{2e^{2x}}{2} e^{2x}$$

$$\int e^x dx = e^x$$

$$\frac{e^{2(1)} - e^{2(0)}}{1}$$

The diagram below shows part of the graph of  $y = \sin 2x$ . The shaded region is between  $x = 0$  and



non-calc

$$P = \frac{2\pi}{b}$$

$$\int \sin x dx = -\cos x$$

$$\left| \left( \frac{-\cos(2x)}{2} \right) \right|$$

$$\left| \frac{-\cos(2(\frac{\pi}{2}))}{2} \right| - \left| \frac{-\cos(2(0))}{2} \right|$$

- a) Write down the period of this function.
- b) Hence or otherwise write down the value of  $m$ .
- c) Find the area of the shaded region.

$$\pi$$

$$\frac{\pi}{2}$$

$$1$$

$$\left| \frac{-1}{2} \right| - \left| \frac{-1}{2} \right|$$

$$\frac{1}{2} + \frac{1}{2}$$

(Total 10 marks)

[Maximum mark: 6]

non-calc

The function  $f$  is given by  $f(x) = 2 \sin(5x - 3)$ .

(a) Find  $f''(x)$ .

(b) Write down  $\int f(x) dx$ .

$\cos x \stackrel{f'(x)}{=} -\sin x$

$\int \sin x dx = -\cos x$

$\sin x \stackrel{f'(x)}{=} \cos x$

$f(x) = 2 \sin(5x - 3)$   
 $2 \cos(5x - 3)$   
 $5 \cdot 2 \cos(5x - 3)$   
 $10 \cos(5x - 3)$   
 $-10 \sin(5x - 3)$   
 $-50 \sin(5x - 3)$

$\int 2 \sin(5x - 3) dx$

$\frac{-2 \cos(5x - 3)}{5} + C$

Let  $f(x) = (3x + 4)^5$ . Find

(a)  $f'(x)$ ;  $(3x+4)^5$   
 $(3x+4)^6$

non-calc

(b)  $\int f(x) dx$ .

$(3x+4)^5$   
 $\frac{(3x+4)^6}{6 \cdot 3} = \frac{(3x+4)^6}{18} + C$

$5(3x+4)^4$

$5 \cdot 3 (3x+4)^4$

$f'(x) = 15(3x+4)^4$



Let  $f(x) = 5 \cos \frac{\pi}{4}x$  and  $g(x) = -0.5x^2 + 5x - 8$ , for  $0 \leq x \leq 9$ .

non-calc ur  
part d)

$$x = \frac{-b}{2a}$$

(5, 4.5) vertex

(a) On the same diagram, sketch the graphs of  $f$  and  $g$ .

(b) Consider the graph of  $f$ . Write down

(i) the  $x$ -intercept that lies between  $x=0$  and  $x=3$ ;  $x=2$

(ii) the period; 8  $p = \frac{2\pi}{b}$

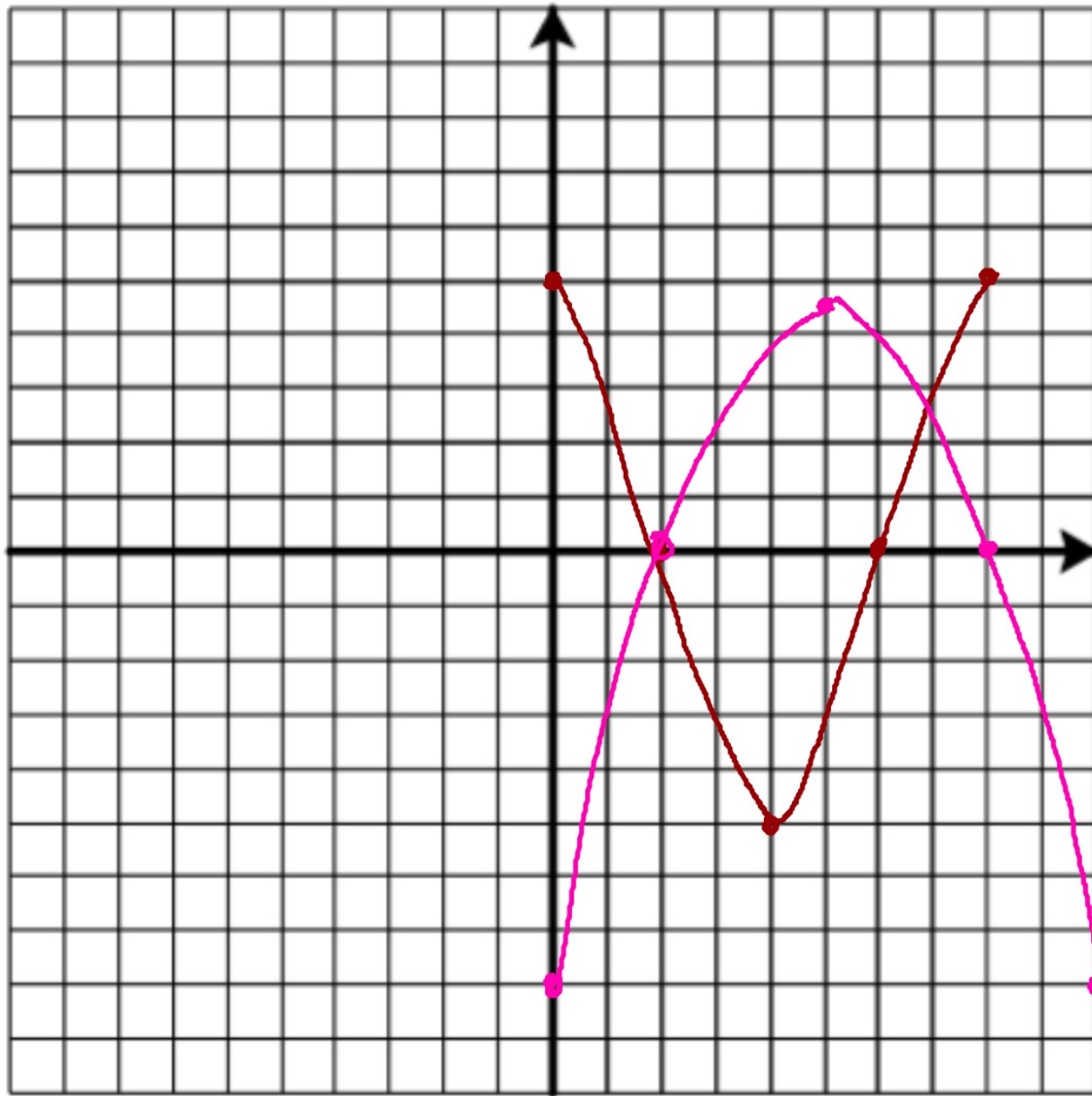
(iii) the amplitude. 5

(c) Consider the graph of  $g$ . Write down

(i) the two  $x$ -intercepts;

(ii) the equation of the axis of symmetry.

(d) Let  $R$  be the region enclosed by the graphs of  $f$  and  $g$ . Find the area of  $R$ .



$f(x)$   
 $g(x)$

