Have you ever seen "Identity?"

PPF : essay planner

PPD : presentation

Today's learning objective:

By the end of class, I will be able to solve circular functions with my trigonometric knowledge.

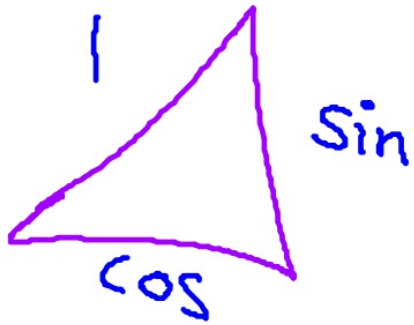
Today's language objective:

Trigonometric identities
Double-angle identities
Pythagorean identity

3.2	Trigonometric identity	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
3.3	Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
	Double angle formulae	$\sin 2\theta = 2\sin\theta\cos\theta$
		$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

non-calc

apples to apples



Q-7. [Maximum marks 7] non-calc

(a) Given that $\cos A = \frac{1}{3}$ and $0 \le A \le \frac{\pi}{2}$, find $\cos 2A$ $= 2\cos^2\theta - 1$

$(\cos\theta)^2 = \cos^2\theta$ ~~$\cos\theta^2$~~

$= 2(\cos\theta)^2 - 1$

$= 2\left(\frac{1}{3}\right)^2 - 1$

$= -\frac{7}{9}$

(b) Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\text{Sin}2B$. $=$

$\cos^2 B + \sin^2 B = 1$

$(\cos B)^2 + (\sin B)^2 = 1$

$(\cos B)^2 + \left(\frac{2}{3}\right)^2 = 1$

$\sqrt{(\cos B)^2} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

$2\sin B\cos B$

$2\left(\frac{2}{3}\right)\cos B$

$2\left(\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right) = \frac{4\sqrt{5}}{9}$

3.2	Trigonometric identity	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
3.3	Pythagorean identity Double angle formulae	$\cos^2\theta + \sin^2\theta = 1$ $\cancel{\sin 2\theta} = 2\sin\theta\cos\theta$ $\cancel{\cos 2\theta} = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

[Maximum mark: 13] non- calc

(a) Let f be the function $f(\theta) = 2\cos 2\theta + 4\cos\theta + 3$, for $-0 \le \theta \le 2\pi$

$\cos\theta = x$ $\quad 2(2\cos^2\theta - 1) + 4\cos\theta + 3$

Show that this function may be written as $(p\cos\theta + 1)^2$.

$f(x) = 4\cos^2\theta \cancel{-2} + 4\cos\theta \cancel{+3} + 1$

Write down the value of p.

$2 = p$ $\quad 4x^2 + 4x + 1$

(4)

~~(b) Draw a rough sketch of the graph $f(\theta)$, $0 \le \theta \le 2\pi$~~ **(3)**

$2x \; + 1$

$2x \; + 1$

$x = -\frac{1}{2}$

$\cos\theta = -\frac{1}{2}$

(c) Consider the equation $f(\theta) = 0$, for $0 \le \theta \le 2\pi$.

Find all values of θ which satisfy this equation.

$\theta = 2\pi/3, \; 4\pi/3$

(2)

(d) Given that $f(\theta) = c$ is satisfied by only two values of θ, find the value of c.

$-1 < c \le 1$

(2)

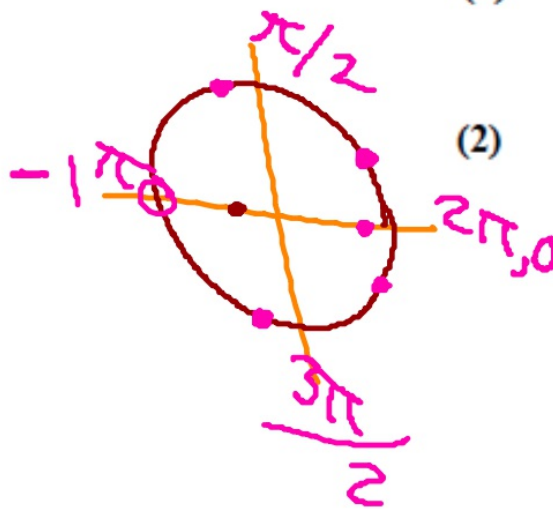
(e) Consider the equation $f(\theta) = 1$, for $-2\pi \le \theta \le 2\pi$.

$4\cos^2\theta + 4\cos\theta = 0$

How many distinct values of $\cos\theta$ satisfy this equation?

(2)

4



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[Maximum mark: 7] non-calc

Given that $\frac{\pi}{2} \le \theta \le \pi$ and that $\cos\theta = -\frac{12}{13}$, find

$\frac{12}{\substack{12\\24\\120\\144}}$

$$\sin^2\theta + (\cos\theta)^2 = 1$$

$$\sin^2\theta + \left(-\frac{12}{13}\right)^2 = 1$$

$$\sqrt{(\sin\theta)^2} = \sqrt{\frac{25}{169}}$$

$$\sin\theta = \cancel{\pm}\frac{5}{13}$$

(a) $\sin\theta$; *[3 marks]*

(b) $\cos 2\theta$; *[3 marks]*

$$\cos^2\theta - \sin^2\theta$$

$$\frac{144}{169} - \frac{25}{169} = \frac{119}{169}$$

(c) $\sin(\theta + \pi)$. *[1 mark]*

$$\boxed{-\frac{5}{13}}$$

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[Maximum mark: 6]

$\sin\theta = x$ $\quad x = -1$ non-calc

(a) Given that $2\sin^2\theta + \sin\theta - 1 = 0$, find the two values for $\sin\theta$. *[4 marks]*

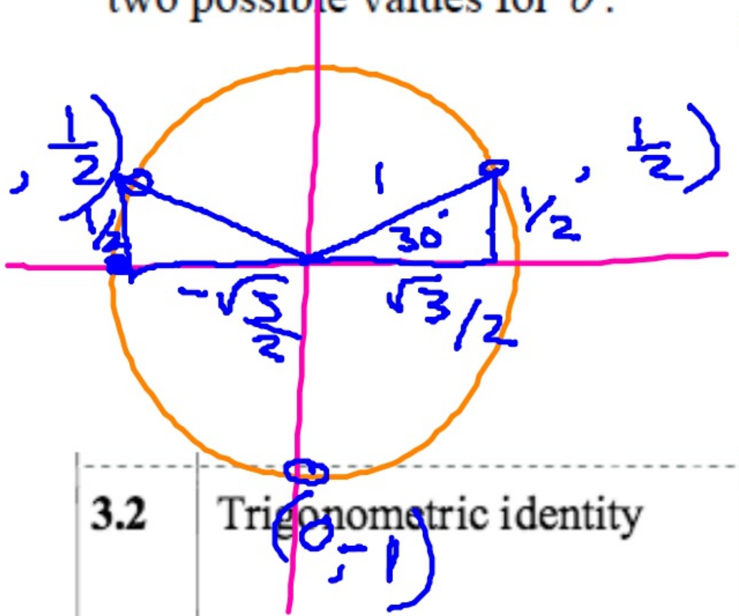
$2x \quad -1$

$x \quad +1$

$x = 1/2$

(b) Given that $0^\circ \le \theta \le 360^\circ$ and that one solution for θ is 30°, find the other two possible values for θ. *[2 marks]*

$\theta = 150^\circ, 270^\circ$



$\sin\theta = \frac{1}{2}$

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5.) Let $f(x) = \sin^3 x + \cos^{\cancel{3}} x \, \cancel{\tan x}\ \frac{\sin x}{\cancel{\cos^2 x}}$, $\frac{\pi}{2} < x < \pi$.

(a) Show that $f(x) = \sin x$.

$\sin^3 x + \cos^2 x \sin x$

(b) Let $\sin x = \frac{2}{3}$. Show that $f(2x) = -\frac{4\sqrt{5}}{9}$.

(To

$\sin x(\underbrace{\sin^2 x + \cos^2 x}_{1})$

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