

Mr. Rogers' current health prognosis:



Can you and your peers switch roles with Mr. Rogers for today's math class?

Today's learning objective:

why is makenna so  
mean to me?  
me = Mitch

By the end of class, I will be able to solve arithmetic sequences and series problems.

Today's language objective:

Arithmetic vs Geometric

$d =$

Sequence vs Series

**1.1**

The  $n^{\text{th}}$  term of an arithmetic sequence

$$u_n = u_1 + (n-1)d$$

The sum of  $n$  terms of an arithmetic sequence

$$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$$

1.) In an arithmetic sequence,  $u_1 = 2$  and  $u_3 = 8$ .

(a) Find  $d$ .

$$2 + (n-1)d \quad \boxed{3} \quad 2 + 3 = 5 + 3 = 8 \quad (2)$$

(b) Find  $u_{20}$ .

$$\boxed{59} \quad 2 + (20-1)3 \quad (19)3 \quad 57 + 2 \quad (2)$$

(c) Find  $S_{20}$ .

$$\frac{20}{2} (2 + 59) \quad 20 + 590 \quad \boxed{610} \quad (2)$$

(Total 6 marks)

2.) In an arithmetic sequence  $u_1 = 7$ ,  $u_{20} = 64$  and  $u_n = 3709$ .

(a) Find the value of the common difference.

$$u_{20} = 64 = 7 + 19d \quad \boxed{3} \quad (3)$$

(b) Find the value of  $n$ .

$$3709 = 7 + (n-1)3 \quad \boxed{1235} \quad (2)$$

(Total 5 marks)

1.1

The  $n^{\text{th}}$  term of an arithmetic sequence

The sum of  $n$  terms of an arithmetic sequence

$$u_n = u_1 + (n-1)d$$

$$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n) \quad \boxed{\phantom{000}}$$

3.) Consider the arithmetic sequence 3, 9, 15, ..., 1353.

why is makenna so mean to me?

(a) Write down the common difference.

$$9 - 3 = 6$$

(1)

(b) Find the number of terms in the sequence.

$$1353 - 3 = 1350 / 6 \quad 1353 = 3 + (n - 1)6 = 226$$

(3)

(c) Find the sum of the sequence.

$$S_n = \frac{226}{2} (3 + 1353) = 153,228$$

(2)

(Total 6 marks)

4.) An arithmetic sequence,  $u_1, u_2, u_3, \dots$ , has  $d = 11$  and  $u_{27} = 263$ .

(a) Find  $u_1$ .

$$S_{50} = \frac{50}{2} (-23 + 516)$$

$$263 = u_1 + (27 - 1)11$$

(2)

(b) (i) Given that  $u_n = 516$ , find the value of  $n$ .

$$S_{50} = 25(493)$$

$$516 = -23 + (n - 1)11$$

(ii) For this value of  $n$ , find  $S_n$ .

$$S_{50} = 12325$$

$$516 = -23 + 11n - 11$$

$$516 = -34 + 11n$$

$$-23 = u_1$$

(4)

(Total 6 marks)

1.1

The  $n^{\text{th}}$  term of an arithmetic sequence

$$u_n = u_1 + (n - 1)d$$

The sum of  $n$  terms of an arithmetic sequence

$$S_n = \frac{n}{2} (2u_1 + (n - 1)d) = \frac{n}{2} (u_1 + u_n)$$

$$550 = 11n \quad n = 50$$

*Naptime phds why: is makenna so mean to me?*

6.) The  $n^{\text{th}}$  term of an arithmetic sequence is given by  $u_n = 5 + 2n$ .

(a) Write down the common difference.

(1)

(b) (i) Given that the  $n^{\text{th}}$  term of this sequence is 115, find the value of  $n$ .

(ii) For this value of  $n$ , find the sum of the sequence.

(5)

(Total 6 marks)

7.) In an arithmetic series, the first term is  $-7$  and the sum of the first 20 terms is 620.

(a) Find the common difference.

$$620 = \frac{20}{2}(2(-7) + (20-1)d)$$

(3)

(b) Find the value of the  $78^{\text{th}}$  term.

(2)

(Total 5 marks)

1.1	<p>The <math>n^{\text{th}}</math> term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>	$u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
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10.) In an arithmetic sequence,  $S_{40} = 1900$  and  $u_{40} = 106$ . Find the value of  $u_1$  and of  $d$ .

(Total 6 marks)

$$\begin{aligned}
 1900 &= 20(2u_1 + (40-1)d) \rightarrow 1900 = 40u_1 + 780d \\
 106 &= u_1 + (40-1)d \quad [106 = u_1 + 39d] \times 40 \\
 106 &= u_1 + 39(3) = 106 = u_1 + 117 \quad -4240 = 40u_1 - 1560d
 \end{aligned}$$

11.) Consider the arithmetic sequence 2, 5, 8, 11, ...

(a) Find  $u_{101}$ .

(b) Find the value of  $n$  so that  $u_n = 152$ .

$$\begin{aligned}
 & \boxed{u_1 = -11} \quad \begin{array}{r} -117 \quad -117 \\ -2340 = -780d \\ \hline -780 \end{array} \quad (3) \\
 & \boxed{d = 3} \quad (3)
 \end{aligned}$$

(Total 6 marks)

1.1	<p>The <math>n^{\text{th}}</math> term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>	$u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
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14.) In an arithmetic sequence  $u_{21} = -37$  and  $u_4 = -3$ .

(a) Find

(i) the common difference;

$$-3 - (-6) = 3$$

(ii) the first term.

$$-2 \times 3 = -6$$

(b) Find  $S_{10}$ .

$$3 + (9) \cdot 3 = 30$$

$$3 - 18 = -15$$

1.1

The  $n^{\text{th}}$  term of an arithmetic sequence

The sum of  $n$  terms of an arithmetic sequence

$$u_n = u_1 + (n-1)d$$

$$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$$

$$-3 = u_1 + (4-1)d$$

$$-37 = u_1 + (21-1)d$$

$$-3 = u_1 + 3d$$

$$-37 = u_1 + 20d$$

$$\times -1$$

$$-3 = u_1 + 3d$$

$$37 = -u_1 - 20d$$

$$37 = -17d$$

$$15 - 95 = -80$$

$$5(-12) = -60$$

(4)

(3)

(Total 7 marks)

20.) (a) Write down the first three terms of the sequence  $u_n = 3n$ , for  $n \geq 1$ .

$$u_1 = 3 \quad u_2 = 6 \quad u_3 = 9$$

(1)

(b) Find

(i)  $\sum_{n=1}^{20} 3n$ ;

$$S_{20} = 10(3 + 60) \rightarrow \boxed{630}$$

(ii)  $\sum_{n=21}^{100} 3n$ .

$$S_{100} = 50(3 + 300) \rightarrow 15150$$

$$S_{20} = 10(3 + 60) \rightarrow 630$$

(5)

(Total 6 marks)

$$\begin{array}{r} 15150 \\ - 630 \\ \hline 14520 \end{array} = \boxed{14,520}$$

1.1	<p>The <math>n^{\text{th}}</math> term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>	$u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
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27.) Let  $S_n$  be the sum of the first  $n$  terms of an arithmetic sequence, whose first three terms are  $u_1, u_2$  and  $u_3$ . It is known that  $S_1 = 7$ , and  $S_2 = 18$ .

(a) Write down  $u_1$ .  $7$

(b) Calculate the common difference of the sequence.

$$18 - 7 = 11 \quad 11 - 7 = 4 = d$$

(c) Calculate  $u_4$ .

$$7 + 4 = 11 + 4 = 15 + 4 = 19 = u_4$$

31.) Gwendolyn added the multiples of 3, from 3 to 3750 and found that

$$3 + 6 + 9 + \dots + 3750 = s.$$

Calculate  $s$ .

$$\begin{aligned} u_n = 3750 &= 3 + 3(n-1) \\ 3750 &= 3n \\ 1250 &= n \end{aligned}$$

$$\begin{aligned} S_{1250} &= 625(3 + 3750) \\ &= 625(3753) \\ S_{1250} &= 2345625 \end{aligned}$$

1.1	<p>The <math>n^{\text{th}}</math> term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>	$u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
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