

Today's learning objective:

By the end of class, I will be able to solve arithmetic sequences and series problems.

Today's language objective:

Arithmetic vs Geometric

$d =$

Sequence vs Series

I will utilize this vocabulary in cooperative groups.

**1.1**The  $n^{\text{th}}$  term of an arithmetic sequence

$$u_n = u_1 + (n-1)d$$

The sum of  $n$  terms of an arithmetic sequence

$$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$$

1.) In an arithmetic sequence,  $u_1 = 2$  and  $u_3 = 8$ . non-calc

(a) Find  $d$ .  $8 = 2 + (3-1)d; 6 = 2d; d = 3$  (2)

(b) Find  $u_{20}$ .  $u_{20} = 2 + (20-1)(3) = 59$  (2)

(c) Find  $S_{20}$ .  $S_{20} = 20/2(2*2 + (20-1)3) = 610$  (2)  
(Total 6 marks)

2.) In an arithmetic sequence  $u_1 = 7, u_{20} = 64$  and  $u_n = 3709$ . calc

(a) Find the value of the common difference.  $64 = 7 + (20-1)d; 57 = 19d$   
 $d = 3$  (3)

(b) Find the value of  $n$ . (2)  
(Total 5 marks)

<b>1.1</b>	The $n^{\text{th}}$ term of an arithmetic sequence	$3709 = 7 + (n-1)3$ $u_n = u_1 + (n-1)d$ $3702 = 3n - 3$
	The sum of $n$ terms of an arithmetic sequence	$3705 = 3n; n = 1,235$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

3.) Consider the arithmetic sequence 3, 9, 15, ..., 1353.

calc

(a) Write down the common difference.

$$d = 6 \quad (1)$$

(b) Find the number of terms in the sequence.

$$1353 = 3 + (n-1)6 \quad (3)$$

(c) Find the sum of the sequence.

$$1350 = 6n - 6 \quad (2)$$

$$1356 = 6n; n = 226 \quad (\text{Total 6 marks})$$

$$S_{226} = 226/2(2 \cdot 3 + (226-1)6) = 153,228$$

4.) An arithmetic sequence,  $u_1, u_2, u_3, \dots$ , has  $d = 11$  and  $u_{27} = 263$ .

calc

(a) Find  $u_1$ .

$$263 = u_1 + (27-1)11; 263 = u_1 + 286$$

(b) (i) Given that  $u_n = 516$ , find the value of  $n$ .

$$u_1 = -23$$

(ii) For this value of  $n$ , find  $S_n$ .

$$S = 50/2(-23 + 516) = 12,325 \quad (4)$$

(Total 6 marks)

1.1	<p>The <math>n^{\text{th}}</math> term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>	$u_n = u_1 + (n-1)d \quad 516 = -23 + (n-1)11$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n) \quad 539 = 11n - 11$ $550 = 11n; n = 50$
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6.) The  $n^{\text{th}}$  term of an arithmetic sequence is given by  $u_n = 5 + 2n$ . **calc**

(a) Write down the common difference.

$u_1 = 7; u_2 = 9; d = 2$  (1)

(b) (i) Given that the  $n^{\text{th}}$  term of this sequence is 115, find the value of  $n$ .

(ii) For this value of  $n$ , find the sum of the sequence.

$115 = 7 + (n-1)2$

$108 = 2n - 2; n = 55; S_{55} = 55/2(7+115) = 3,355$

(5)  
(Total 6 marks)

7.) In an arithmetic series, the first term is  $-7$  and the sum of the first 20 terms is 620. **calc**

(a) Find the common difference.

$620 = 20/2(2 \cdot -7 + (20-1)d)$  (3)

(b) Find the value of the 78<sup>th</sup> term.

$620 = 10(-14 + 19d)$

$62 = -14 + 19d$

(2)  
(Total 5 marks)

$76 = 19d; d = 4$

$u_{78} = -7 + (78-1)4 = 301$

term of an  
arithmetic sequence

$u_n = u_1 + (n-1)d$

sum of  $n$  terms of an  
arithmetic sequence

$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

10.) In an arithmetic sequence,  $S_{40} = 1900$  and  $u_{40} = 106$ . Find the value of  $u_1$  and of  $d$ .

(Total 6 marks)

$$106 = u_1 + (40-1)d; \quad 106 = -11 + 39d$$

$$106 = u_1 + 39d \quad 117 = 39d; d=3 \quad \text{calc}$$

$$1900 = 40/2 (u_1 + 106)$$

$$1900 = 20u_1 + 2120; -220 = 20u_1; u_1 = -11$$

11.) Consider the arithmetic sequence 2, 5, 8, 11, ....

(a) Find  $u_{101}$ .  $d = 3; u_{101} = 2 + (101-1)3; u_{101} = 302$  (3)

(b) Find the value of  $n$  so that  $u_n = 152$ .

$$152 = 2 + (n-1)3$$

$$150 = 3n - 3$$

$$153 = 3n; n = 51$$

(3)  
(Total 6 marks)

calc

<b>1.1</b>	The $n^{\text{th}}$ term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of $n$ terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

14.) In an arithmetic sequence  $u_{21} = -37$  and  $u_4 = -3$ .

(a) Find  $d = -2$

(i) the common difference;

(ii) the first term.  $u_1 = 3$

(b) Find  $S_{10}$ .

$$S_{10} = 10/2(2*3 + (10-1)*-2) \quad (\text{Total 7 m})$$

$$S_{10} = 5(6 - 18)$$

$$S_{10} = -60$$

**1.1**

The  $n^{\text{th}}$  term of an arithmetic sequence

The sum of  $n$  terms of an arithmetic sequence

$$u_n = u_1 + (n-1)d$$

$$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$$

20.) (a) Write down the first three terms of the sequence  $u_n = 3n$ , for  $n \geq 1$ .

(1)

(b) Find

(i)  $\sum_{n=1}^{20} 3n$ ;

(ii)  $\sum_{n=21}^{100} 3n$ .

(5)

(Total 6 marks)

<b>1.1</b>	The $n^{\text{th}}$ term of an arithmetic sequence	$u_n = u_1 + (n - 1)d$
	The sum of $n$ terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n - 1)d) = \frac{n}{2}(u_1 + u_n)$

27.) Let  $S_n$  be the sum of the first  $n$  terms of an arithmetic sequence, whose first three terms are  $u_1$ ,  $u_2$  and  $u_3$ . It is known that  $S_1 = 7$ , and  $S_2 = 18$ .

- (a) Write down  $u_1$ .
- (b) Calculate the common difference of the sequence.
- (c) Calculate  $u_4$ .

31.) Gwendolyn added the multiples of 3, from 3 to 3750 and found that

$$3 + 6 + 9 + \dots + 3750 = s.$$

Calculate  $s$ .

$$u_n = 3750 = 3 + 3(n-1)$$

$$3750 = 3n$$

$$1250 = n$$

$$S_{1250} = 625(3 + 3750)$$

$$625(3753)$$

$$S_{1250} = 2345625$$

<b>1.1</b>	<p>The <math>n^{\text{th}}</math> term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>	$u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
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