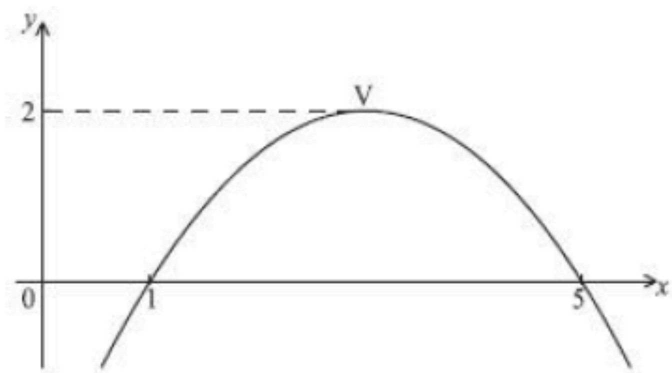
87.) Part of the graph of the function $y = d(x-m)^2 + p$ is given in the diagram below.

The x-intercepts are $(1, 0)$ and $(5, 0)$. The vertex is $V(m, 2)$.



(a) Write down the value of

(i) m;

(ii) p.

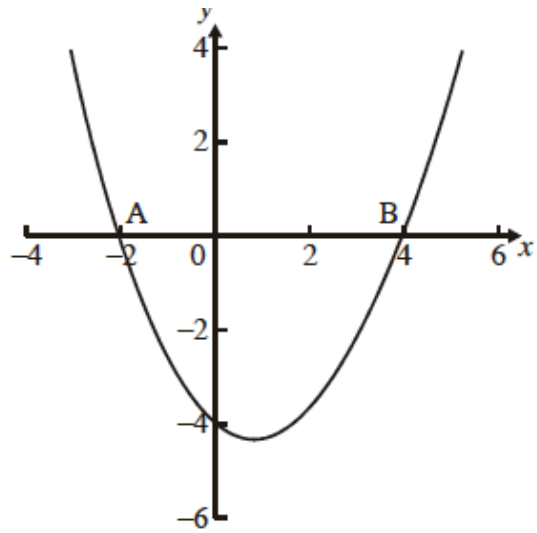
(b) Find d.

(Total 6 marks)

172.) Let $f(x) = \sqrt{x}$, and $g(x) = 2^x$. Solve the equation

$(f^{-1} \circ g)(x) = 0.25$.

99.) The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram below.



(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$.

(5)

(b) (i) Find $\frac{dy}{dx}$.

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P.

(4)

(c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.

(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again.

160.) $f(x) = 4\sin\left(3x + \frac{\pi}{2}\right)$.

For what values of k will the equation $f(x) = k$ have no solutions?

NAME:___________________________________ **Paper Preview: f(x)'s & Sequences DATE: 03/04/16**

167.) The function f is given by

$$f(x) = \frac{2x+1}{x-3}, x \in \mathbb{R}, x \neq 3.$$

(a) (i) Show that $y = 2$ is an asymptote of the graph of $y = f(x)$. **(2)**

(ii) Find the vertical asymptote of the graph. **(1)**

(iii) Write down the coordinates of the point P at which the asymptotes intersect. **(1)**

(b) Find the points of intersection of the graph and the axes. **(4)**

(c) Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines. **(4)**

(d) Show that $f'(x) = \frac{-7}{(x-3)^2}$ and hence find the equation of the tangent at the point S where $x = 4$. **(6)**

(e) The tangent at the point T on the graph is parallel to the tangent at S.

Find the coordinates of T. **(5)**

(f) Show that P is the midpoint of [ST].

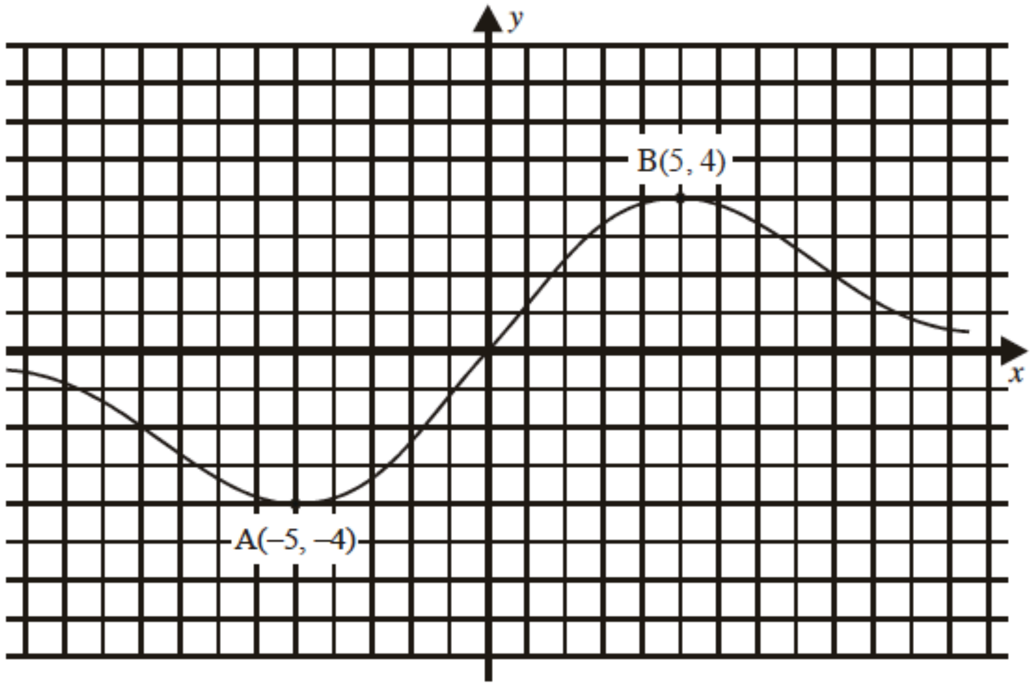
150.) A group of ten leopards is introduced into a game park. After t years the number of leopards, N, is modelled by $N = 10\,e^{0.4t}$.

(a) How many leopards are there after 2 years?

(b) How long will it take for the number of leopards to reach 100? Give your answers to an appropriate degree of accuracy.

Give your answers to an appropriate degree of accuracy.

152.) The diagram shows the graph of $y = f(x)$, with the x-axis as an asymptote.



(a) On the same axes, draw the graph of $y = f(x + 2) - 3$, indicating the coordinates of the images of the points A and B.

(b) Write down the equation of the asymptote to the graph of $y = f(x + 2) - 3$.

33.) Arturo goes swimming every week. He swims 200 metres in the first week. Each week he swims 30 metres more than the previous week. He continues for one year (52 weeks).

(a) How far does Arturo swim in the final week?

(b) How far does he swim altogether?

34.) The diagrams below show the first four squares in a sequence of squares which are subdivided in half. The area of the shaded square A is $\frac{1}{4}$.

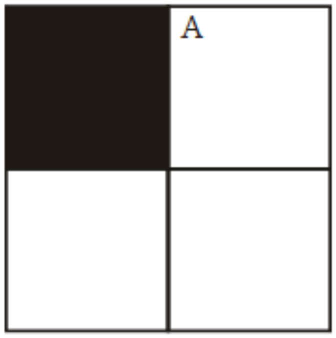


Diagram 1

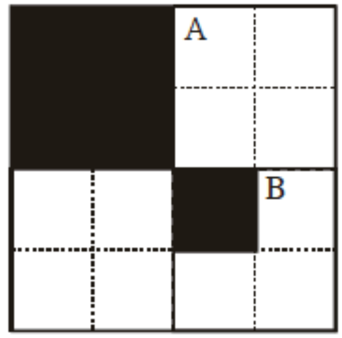


Diagram 2

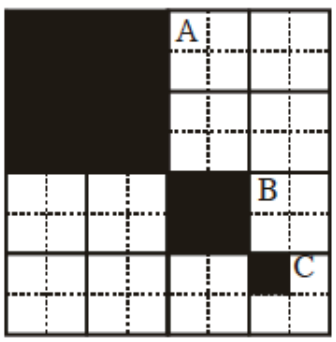


Diagram 3

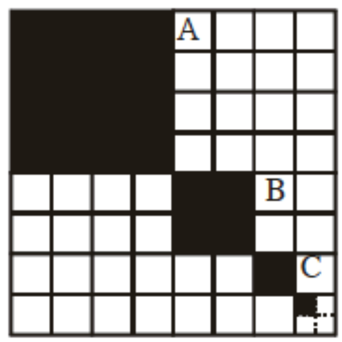


Diagram 4

(a) (i) Find the area of square B and of square C.

(ii) Show that the areas of squares A, B and C are in geometric progression.

(iii) Write down the common ratio of the progression.

(5)

(b) (i) Find the **total** area shaded in diagram 2.

(ii) Find the **total** area shaded in the 8^{th} diagram of this sequence.
Give your answer correct to six significant figures.

(4)

(c) The dividing and shading process illustrated is continued indefinitely.
Find the total area shaded.

(2)
(Total 11 marks)

87.) (a) (i) $m = 3$ A2 N2

(ii) $p = 2$ A2 N2

(b) Appropriate substitution M1

$eg\ 0 = d(1-3)^2 + 2, 0 = d(5-3)^2 + 2, 2 = d(3-1)(3-5)$

$d = -\frac{1}{2}$ A1 N1

[6]

172.) $x = g^{-1}(f(0.25))$ (M1)

$= \log_2((0.25)^{1/2})$ (A1)

$= \log_2\left(\frac{1}{2}\right)$ (A1)

$= -1$ (A1)

OR

$f^{-1}(x) = x^2$ (M1)

$= (f^{-1} \circ g)(x) = f^{-1}(2^x) = 2^{2x}$ (M1)

Therefore, $2^{2x} = 0.25 = 2^{-2}$ (M1)

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$ (A1) (C4)

[4]

99.) (a) (i) $p=-2 \quad q=4$ (or $p=4, q=-2$) (A1)(A1) (N1)(N1)

(ii) $y=a(x+2)(x-4)$

$8=a(6+2)(6-4)$ (M1)

$8=16a$

$a=\frac{1}{2}$ (A1) (N1)

(iii) $y=\frac{1}{2}(x+2)(x-4)$

$y=\frac{1}{2}(x^2-2x-8)$

$y=\frac{1}{2}x^2-x-4$ (A1) (N1)5

(b) (i) $\frac{dy}{dx}=x-1$ (A1) (N1)

(ii) $x-1=7$ (M1)

$x=8, y=20$ (P is $(8, 20)$) (A1)(A1) (N2)4

(c) (i)when $x = 4$, gradient of tangent is $4 - 1 = 3$ (may be implied)(A1)

gradient of normal is $-\frac{1}{3}$ (A1)

$y-0=-\frac{1}{3}(x-4) \left(y=-\frac{1}{3}x+\frac{4}{3}\right)$ (A1) (N3)

(ii) $\frac{1}{2}x^2-x-4=-\frac{1}{3}x+\frac{4}{3}$ (or sketch/graph) (M1)

$\frac{1}{2}x^2-\frac{2}{3}x-\frac{16}{3}=0$

$3x^2-4x-32=0$ (may be implied) (A1)

$(3x+8)(x-4)=0$

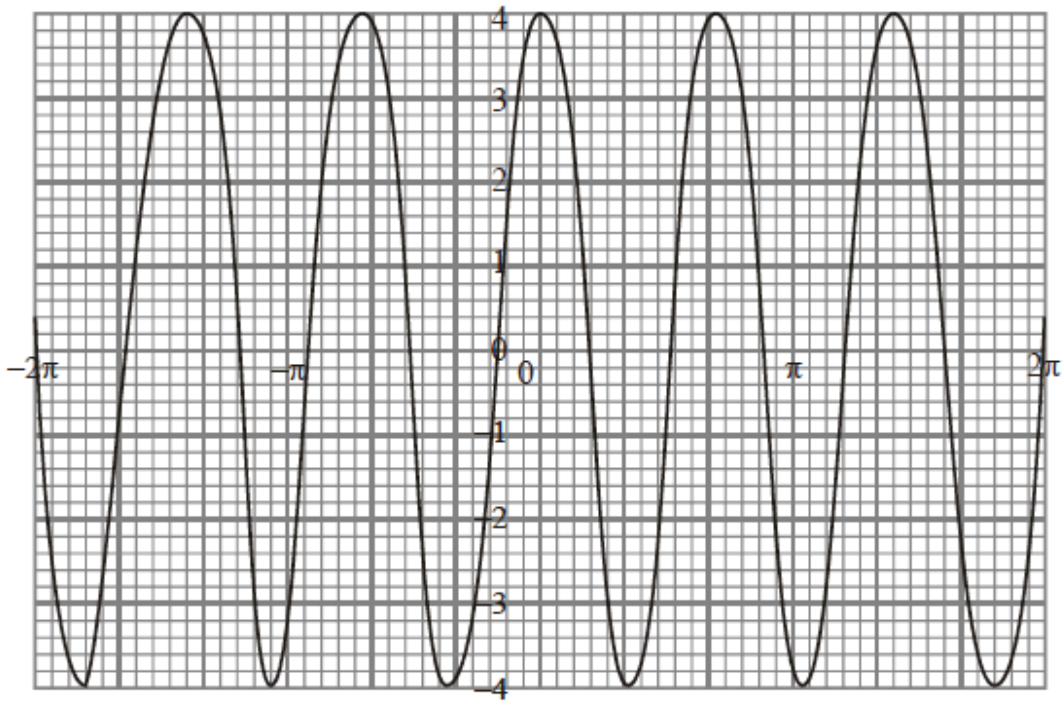
$x=-\frac{8}{3}$ or $x=4$

$x=-\frac{8}{3}$ (-2.67) (A1) (N2)6

[15]

160.) From sketch of graph $y = 4\sin\left(3x + \frac{\pi}{2}\right)$ (M2)

or by observing $|\sin \square| \le 1$.
$k > 4, k < -4$ (A1)(A1) (C2)(C2)



[4]

167.) (a) (i) $f(x) = \frac{2x+1}{x-3}$

$= 2 + \frac{7}{x-3}$ by division or otherwise (M1)
Therefore as $|x| \to \infty$ $f(x) \to 2$ (A1)
$\Rightarrow y = 2$ is an asymptote (AG)

OR $\lim_{x\to\infty} \frac{2x+1}{x-3} = 2$ (M1)(A1)
$\Rightarrow y = 2$ is an asymptote (AG)

OR make x the subject
$yx - 3y = 2x + 1$
$x(y - 2) = 1 + 3y$ (M1)
$x = \frac{1+3y}{y-2}$ (A1)
$\Rightarrow y = 2$ is an asymptote (AG)

Note: Accept inexact methods based on the ratio of the coefficients of x.

(ii) Asymptote at $x = 3$ (A1)

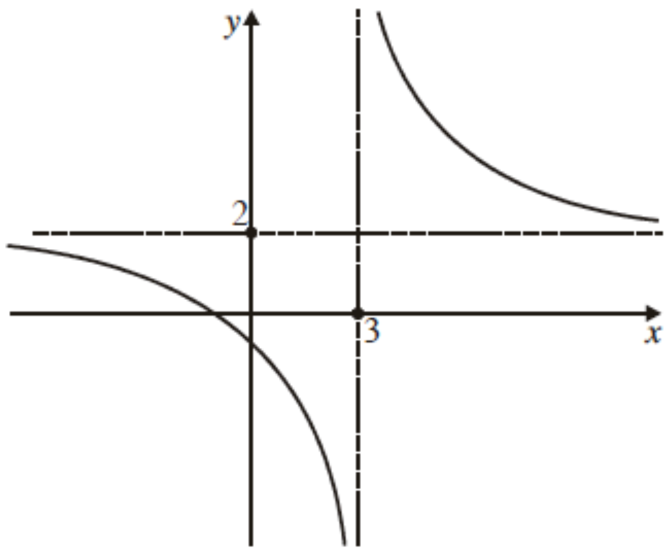
(iii) $P(3, 2)$ (A1) 4

(b) $f(x) = 0 \Rightarrow x = -\frac{1}{2}$ $\left(-\frac{1}{2}, 0\right)$ (M1)(A1)

$x = 0 \Rightarrow f(x) = -\frac{1}{3}$ $\left(0, -\frac{1}{3}\right)$ (M1)(A1) 4

***Note:** These do not have to be in coordinate form.*

(c)



(A4) 4

Note: *Asymptotes (A1)*
Intercepts (A1)
"Shape" (A2).

(d) $f'(x) = \frac{(x-3)(2) - (2x+1)}{(x-3)^2}$ (M1)

$= \frac{-7}{(x-3)^2}$ (A1)

= Slope at any point

Therefore slope when $x = 4$ is -7 (A1)
And $f(4) = 9$ ie $S(4, 9)$ (A1)
$\Rightarrow$ Equation of tangent: $y - 9 = -7(x - 4)$ (M1)
$7x + y - 37 = 0$ (A1) 6

(e) at T, $\frac{-7}{(x-3)^2} = -7$ (M1)

$\Rightarrow (x-3)^2 = 1$ (A1)
$x - 3 = \pm 1$ (A1)
$\left.\begin{array}{l} x = 4 \text{ or } 2 \\ y = 9 \text{ or } -5 \end{array}\right\}$ $S(4,9)$, $T(2,-5)$ (A1)(A1) 5

(f) Midpoint $[ST] = \left(\frac{4+2}{2}, \frac{9-5}{2}\right)$

$= (3, 2)$
= point P (A1) 1

[24]

150.) (a) At $t = 2, N = 10e^{0.4(2)}$ (M1)
$N = 22.3$ (3 sf)

Number of leopards = 22 (A1)

(b) If $N = 100$, then solve $100 = 100e^{0.4t}$

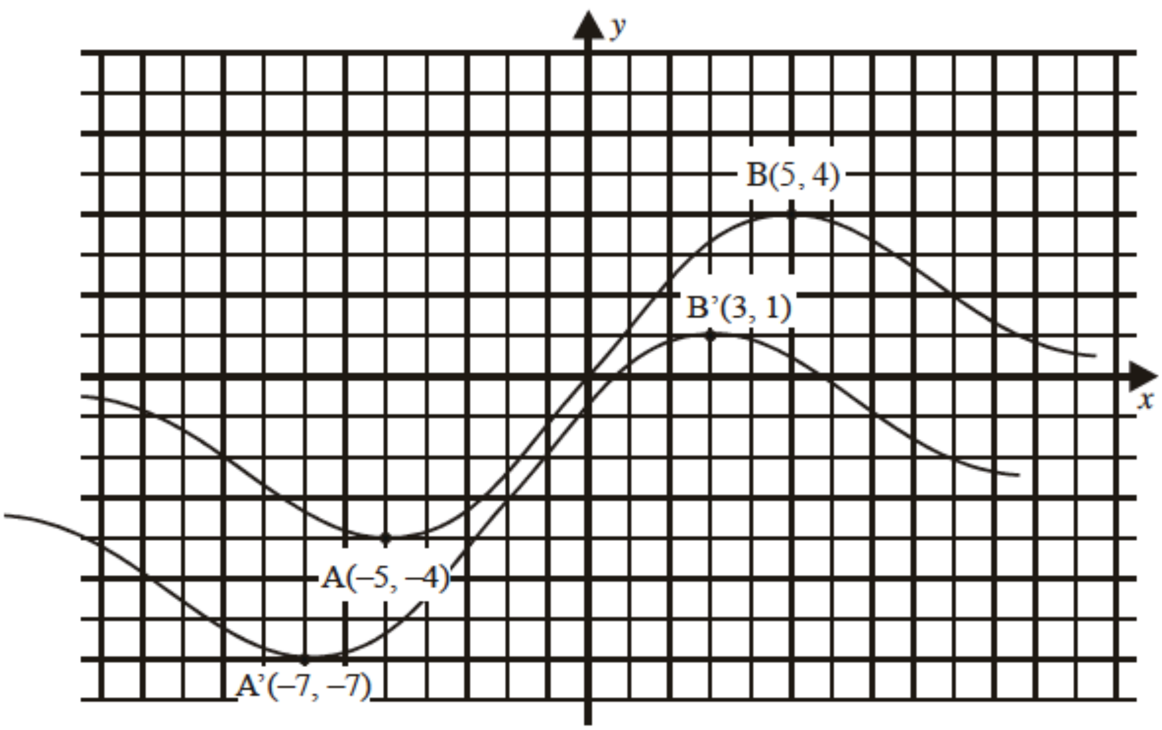
$10 = e04^{t}$

$\ln 10 = 0.4t$

$t = \dfrac{\ln 10}{0.4} \sim 5.76$ years (3 sf) (A1)

[4]

152.) (a) Correct vertical shift (A1)
Coordinates of the images (see diagram) (A1) (A1)



(b) Asymptote: $y = -3$ (A1)

[4]

33.) Arithmetic sequence (M1)

$a = 200$ $d = 30$ (A1)

(a) Distance in final week $= 200 + 51 \times 30$ (M1)

$= 1730$ m (A1) (C3)

(b) Total distance $= \frac{52}{2}[2.200 + 51.30]$ (M1)

$= 50180$ m (A1) (C3)

Note: *Penalize once for absence of units ie award A0 the first time units are omitted, A1 the next time.*

[6]

34.) (a) (i) Area B $= \frac{1}{16}$, area C $= \frac{1}{64}$ (A1)(A1)

(ii) $\frac{\frac{1}{16}}{\frac{1}{4}} = \frac{1}{4}$ $\frac{\frac{1}{64}}{\frac{1}{16}} = \frac{1}{4}$ (Ratio is the same.) (M1)(R1)

(iii) Common ratio $= \frac{1}{4}$ (A1) 5

(b) (i)Total area (S_2) $= \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$ $= (= 0.3125)$ (0.313, 3 sf) (A1)

(ii) Required area $= S_8 = \dfrac{\frac{1}{4}\left(1 - \left(\frac{1}{4}\right)^8\right)}{1 - \frac{1}{4}}$ (M1)

$= 0.333328\ 2(471...)$ (A1)

$= 0.333328$ (6 sf) (A1) 4

Note: *Accept result of adding together eight areas correctly.*

(c) Sum to infinity $= \dfrac{\frac{1}{4}}{1 - \frac{1}{4}}$ (A1)

$= \frac{1}{3}$ (A1) 2

[11]