NAME:_____ DATE: 04/02/2018

ASSIGNMENT: Trains and Integral Calculus
DIRECTIONS: Follow the units if you need to know whether to differentiate or integrate.

INTEGRATION ----->
Acceleration = ms^{-2} **Velocity = ms^{-1}** **Displacement = m**

DIFFERENTIATION ----->
Displacement = m **Velocity = ms^{-1}** **Acceleration = ms^{-2}**

17.) *In this question s represents displacement in metres and t represents time in seconds.*

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t.

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t.

(6)

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

(ii) Hence show that $a = \frac{8}{5}$.

(6)

(c) For a different train N, the value of a is 4.
Show that this train will stop **before** it reaches the station.

(5)
(Total 17 marks)

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Answer key (show all calculations for full marks)

Note: In this question, do not penalize absence of units.

(a) (i) $s = \int (40 - at)\mathrm{d}t$ (M1)

$s = 40t - \frac{1}{2}at^2 + c$ (A1)(A1)

substituting $s = 100$ when $t = 0$ ($c = 100$) (M1)

$s = 40t - \frac{1}{2}at^2 + 100$ A1 N5

(ii) $s = 40t - \frac{1}{2}at^2$ A1 N1

(b) (i) stops at station, so $v = 0$ (M1)

$t = \frac{40}{a}$ (seconds) A1 N2

(ii) evidence of choosing formula for s from (a) (ii) (M1)

substituting $t = \frac{40}{a}$ (M1)

e.g. $40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$

setting up equation M1

e.g. $500 = s$, $500 = 40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$, $500 = \frac{1600}{a} - \frac{800}{a}$

evidence of simplification to an expression which obviously leads to $a = \frac{8}{5}$ A1

e.g. $500a = 800$, $5 = \frac{8}{a}$, $1000a = 3200 - 1600$

$a = \frac{8}{5}$ AGN0

(c) **METHOD 1**

$v = 40 - 4t$, stops when $v = 0$

$40 - 4t = 0$ (A1)

$t = 10$ A1

substituting into expression for s M1

$s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$

$s = 200$ A1

since $200 < 500$ (allow ***FT*** on their s, if $s < 500$) R1

train stops before the station AGN0

METHOD 2

from (b) $t = \frac{40}{4} = 10$ A2

substituting into expression for s

e.g. $s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$ M1

$s = 200$ A1

since $200 < 500$, R1

train stops before the station AGN0

METHOD 3

a is deceleration A2

$4 > \frac{8}{5}$ A1

so stops in shorter time (A1)

so less distance travelled R1

so stops before station AGN0