

What's the probability that class will be an enjoyable experience given that you're in math?

Math

.

If  $\alpha = 0.4$  and  $\beta = 0.65$ , find the value of  $\Omega$  if the following statement is true:

$$4\alpha\Omega = \frac{\beta}{3}$$

## Today's learning objective:

By the end of class, I will be able to manipulate probability formulas algebraically to solve problems.

## Today's language objective:

You will be able to use the phrase "given that" in context appropriately with a neighbor.

5.5	Probability of an event $A$  Complementary events	$P(A) = \frac{n(A)}{n(U)}$  $P(A) + P(A') = 1$
5.6	Combined events  Mutually exclusive events  Conditional probability  Independent events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B)$ $P(A \cap B) = P(A) P(B   A)$ $P(A \cap B) = P(A) P(B)$
5.7	Expected value of a discrete random variable $X$	$E(X) = \mu = \sum_x x P(X = x)$
5.8	Binomial distribution  Mean  Variance	$X \sim B(n, p) \Rightarrow P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, r = 0, 1, \dots, n$ $E(X) = np$ $\text{Var}(X) = np(1-p)$
5.9	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

33.) Let  $A$  and  $B$  be independent events, where  $P(A) = 0.6$  and  $P(B) = x$ .

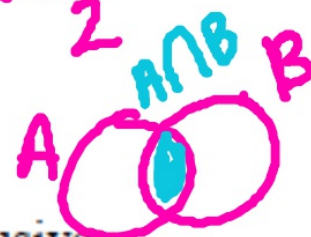
(a) Write down an expression for  $P(A \cap B)$ .  $= 0.6x$

(b) Given that  $P(A \cup B) = 0.8$ .  $= 0.6 + x - 0.6x$

(i) find  $x$ :  $0.8 = 0.6 + 0.4x$   $x = \frac{1}{2}$

(ii) find  $P(A \cap B)$ .

$0.3$



(c) Hence, explain why  $A$  and  $B$  are **not** mutually exclusive.

Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
<del>Mutually exclusive events</del>	<del><math>P(A \cup B) = P(A) + P(B)</math></del>
<del>Conditional probability</del>	<del><math>P(A \cap B) = P(A)P(B A)</math></del>
Independent events	$P(A \cap B) = P(A)P(B)$

25.) Consider the independent events  $A$  and  $B$ . Given that  $P(B) = 2P(A)$ , and  $P(A \cup B) = 0.52$ , find  $P(B)$ .

$$P(A \cup B) = 0.52 = A + 2A - 2A^2$$

$$0.52 = 3A - 2A^2$$

(To

$$P(A \cap B) = P(A) \cdot 2P(A) \quad 0 = 3A - 2A^2 - 0.52$$

$$P(A \cap B) = A \cdot 2A = 2A^2$$

~~$$x = 1.3 \quad A = x = .2$$~~

$$B = 0.4$$

Combined events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

~~Mutually exclusive events~~

~~$$P(A \cup B) = P(A) + P(B)$$~~

~~Conditional probability~~

~~$$P(A \cap B) = P(A)P(B|A)$$~~

Independent events

$$P(A \cap B) = P(A)P(B)$$

44.) Consider the events  $A$  and  $B$ , where  $P(A) = \frac{2}{5}$ ,  $P(B) = \frac{1}{4}$  and  $P(A \cup B) = \frac{7}{8}$ .

(a) Write down  $P(B) = \frac{3}{4}$

(b) Find  $P(A \cap B)$ .  $\frac{7}{8} = \frac{2}{5} + \frac{3}{4} - P(A \cap B)$

(c) Find  $P(A | B)$ .  $\frac{11}{40} = P(A \cap B)$

$$\frac{11}{40} = \frac{3}{4} \cdot P(A | B)$$

$$\frac{11}{30} = P(A | B)$$

Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
<del>Mutually exclusive events</del>	<del><math>P(A \cup B) = P(A) + P(B)</math></del>
Conditional probability	$P(A \cap B) = P(A)P(B   A)$ $P(B) \cdot (A   B)$
<del>Independent events</del>	<del><math>P(A \cap B) = P(A)P(B)</math></del>

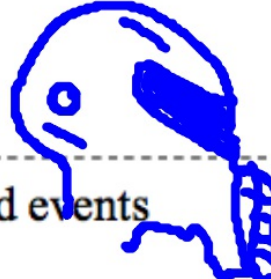
54.) Let  $A$  and  $B$  be independent events such that  $P(A) = 0.3$  and  $P(B) = 0.8$ .

(a) Find  $P(A \cap B)$ . = 0.24

(b) Find  $P(A \cup B)$ . = 0.86

(c) Are  $A$  and  $B$  mutually exclusive? Justify your answer.

$$0.3 + 0.8 = 1.1$$

$$0.3 + 0.8 > 1$$


Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
<del>Mutually exclusive events</del>	<del><math>P(A \cup B) = P(A) + P(B)</math></del>
<del>Conditional probability</del>	<del><math>P(A \cap B) = P(A)P(B A)</math></del>
Independent events	$P(A \cap B) = P(A)P(B)$

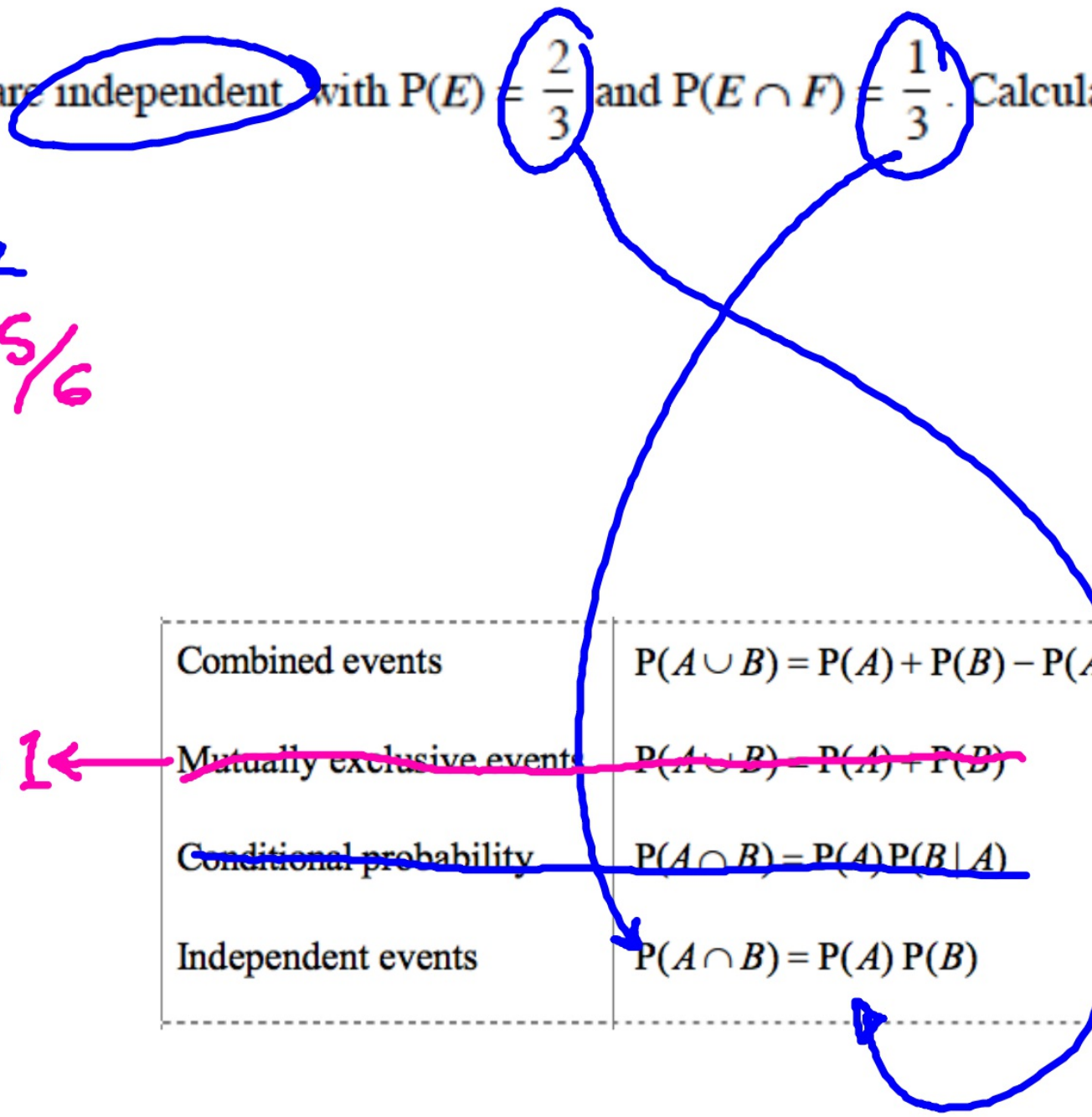


51.) Events  $E$  and  $F$  are independent with  $P(E) = \frac{2}{3}$  and  $P(E \cap F) = \frac{1}{3}$ . Calculate:

- (a)  $P(F)$ ;  $\frac{1}{2}$   
 (b)  $P(E \cup F)$ ;  $\frac{5}{6}$

$\frac{1}{2} + \frac{2}{3} > 1$  ←

Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
<del>Mutually exclusive events</del>	<del><math>P(A \cup B) = P(A) + P(B)</math></del>
<del>Conditional probability</del>	<del><math>P(A \cap B) = P(A)P(B A)</math></del>
Independent events	$P(A \cap B) = P(A)P(B)$



The events  $A$  and  $B$  are independent such that  $P(B) = 3P(A)$  and  $P(A \cup B) = 0.68$ . Find

RTFQ

$$P(B) = 0.6$$

Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
<del>Conditional probability</del>	<del><math>P(A \cap B) = P(A)P(B A)</math></del>
Independent events	$P(A \cap B) = P(A)P(B)$

75.) Let  $A$  and  $B$  be events such that  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{3}{4}$  and  $P(A \cup B) = \frac{7}{8}$ .

- (a) Calculate  $P(A \cap B)$ .
- (b) Calculate  $P(A|B)$ .
- (c) Are the events  $A$  and  $B$  independent? Give a reason for your answer.

Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
Conditional probability	$P(A \cap B) = P(A)P(B A)$
Independent events	$P(A \cap B) = P(A)P(B)$

89.) For events  $A$  and  $B$ , the probabilities are  $P(A) = \frac{3}{11}$ ,  $P(B) = \frac{4}{11}$ .

Calculate the value of  $P(A \cap B)$  if

(a)  $P(A \cup B) = \frac{6}{11}$ ;

(b) events  $A$  and  $B$  are independent.

Combined events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$

Conditional probability

$$P(A \cap B) = P(A)P(B|A)$$

Independent events

$$P(A \cap B) = P(A)P(B)$$

49.) Two restaurants, *Center* and *New*, sell fish rolls and salads.

Let  $F$  be the event a customer chooses a fish roll.

Let  $S$  be the event a customer chooses a salad.

Let  $N$  be the event a customer chooses neither a fish roll nor a salad.

In the *Center* restaurant  $P(F) = 0.31$ ,  $P(S) = 0.62$ ,  $P(N) = 0.14$ .

- (a) Show that  $P(F \cap S) = 0.07$ .
- (b) Given that a customer chooses a salad, find the probability the customer also chooses a fish roll.
- (c) Are  $F$  and  $S$  independent events? Justify your answer.

At *New* restaurant,  $P(N) = 0.14$ . Twice as many customers choose a salad as choose a fish roll. Choosing a fish roll is **independent** of choosing a salad.

- (d) Find the probability that a fish roll is chosen.

