NAME:______________________________________ DATE: 09/11A-12B/2017

<u>ASSIGNMENT</u>: Unit Vectors, Magnitude, Parallel and Perpendicular Vectors
<u>DIRECTIONS</u>:

To find magnitude: it's like the Pythagorean theorem: $\sqrt{(v_1^2 + v_2^2 + v_3^2)}$
To find unit vector: divide the vector by the magnitude
Parallel vectors have direction vectors that are scalar multiples $d_1 \cdot d_2 = |d_1||d_2|$
Perpendicular vectors have direction vectors whose product is zero

31.) The position vector of point A is $2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and the position vector of point B is $4\boldsymbol{i} - 5\boldsymbol{j} + 21\boldsymbol{k}$.

(a) (i) Show that $\overrightarrow{AB} = 2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k}$.

(ii) Find the unit vector $\boldsymbol{u}$ in the direction of $\overrightarrow{AB}$.

(iii) Show that $\boldsymbol{u}$ is perpendicular to $\overrightarrow{OA}$.

(6)

Let S be the midpoint of [AB]. The line L_1 passes through S and is parallel to $\overrightarrow{OA}$.

(b) (i) Find the position vector of S.

(ii) Write down the equation of L_1.

(4)

The line L_2 has equation $\boldsymbol{r} = (5\boldsymbol{i} + 10\boldsymbol{j} + 10\boldsymbol{k}) + s(-2\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k})$.

(c) Explain why L_1 and L_2 are not parallel.

(2)

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31.) (a) (i) Evidence of subtracting all three components in the correct order M1

eg $\vec{AB} = \vec{OB} - \vec{OA} = (4i - 5j + 21k) - (2i + 3j + k)$

$= 2i - 8j + 20k$ AG N0

(ii) $|\vec{AB}| = \sqrt{2^2 + (-8)^2 + 20^2}$ $\left(= \sqrt{468} = 6\sqrt{13} = 2\sqrt{117} = 21.6\right)$ (A1)

$u = \frac{1}{\sqrt{468}}(2i - 8j + 20k)$ A1 N2

$\left(= \frac{2}{\sqrt{468}}i - \frac{8}{\sqrt{468}}j + \frac{20}{\sqrt{468}}k, 0.0925i - 0.370j + 0.925k, \text{etc.}\right)$

(iii) If the scalar product is zero, the vectors are perpendicular. R1

Note: Award R1 for stating the relationship between the scalar product and perpendicularity, seen anywhere in the solution.

Finding an appropriate scalar product $\left(u \bullet \vec{OA} \text{ or } \vec{AB} \bullet \vec{OA}\right)$ M1

eg $u \bullet \vec{OA} = \left(\frac{2}{\sqrt{468}}\right) \times 2 + \left(\frac{-8}{\sqrt{468}}\right) \times 3 + \left(\frac{20}{\sqrt{468}}\right) \times 1$

$\left(= \frac{4 - 24 + 20}{\sqrt{468}}\right)$

$\vec{AB} \bullet \vec{OA} = 2 \times 2 + (-8) \times 3 + 20 \times 1$

$u \bullet \vec{OA} = 0$ or $\vec{AB} \bullet \vec{OA} = 0$ A1 N0

(b) (i) **EITHER**

$S\left(\frac{2+4}{2}, \frac{3-5}{2}, \frac{1+21}{2}\right)$ (M1)(A1)

Therefore, $\vec{OS} = 3i - j + 11k$ (accept $(3, -1, 11)$) A1 N3

OR

$\vec{OS} = \vec{OA} + \frac{1}{2}\vec{AB}$ (M1)

$= (2i + 3j + k) + \frac{1}{2}(2i + 8j + 20k)$ (A1)

$\vec{OS} = 3i - j + 11k$ A1 N3

(ii) $L_1 : r = (3i - j + 11k) + t(2i + 3j + 1k)$ A1 N1

(c) Using direction vectors (*eg* $2i + 3j + 1k$ and $-2i + 5j - 3k$) (M1)

Valid explanation of why L_1 is not parallel to L_2 R1 N2

eg. Direction vectors are not scalar multiples of each other.
Angle between the direction vectors is not zero or 180.
Finding the angle

$d_1 \bullet d_2 \neq |d_1||d_2|$.

Note: *Award R0 for "direction vectors are not equal".*