

## Exams:

- 1) Good job!
- 2) If you don't like your grade, please correct the exam in tutorials and retake afterward
- 3) Maximum retake grade is a 70%

Can you sketch the following scenario on your desk?

\*A Nerf arrow is shot at a trajectory to where the arrow is tangent to the top of your head.



How does this situation with the Nerf arrow relate to tangent lines, gradients, and Differential Calculus?

Slope

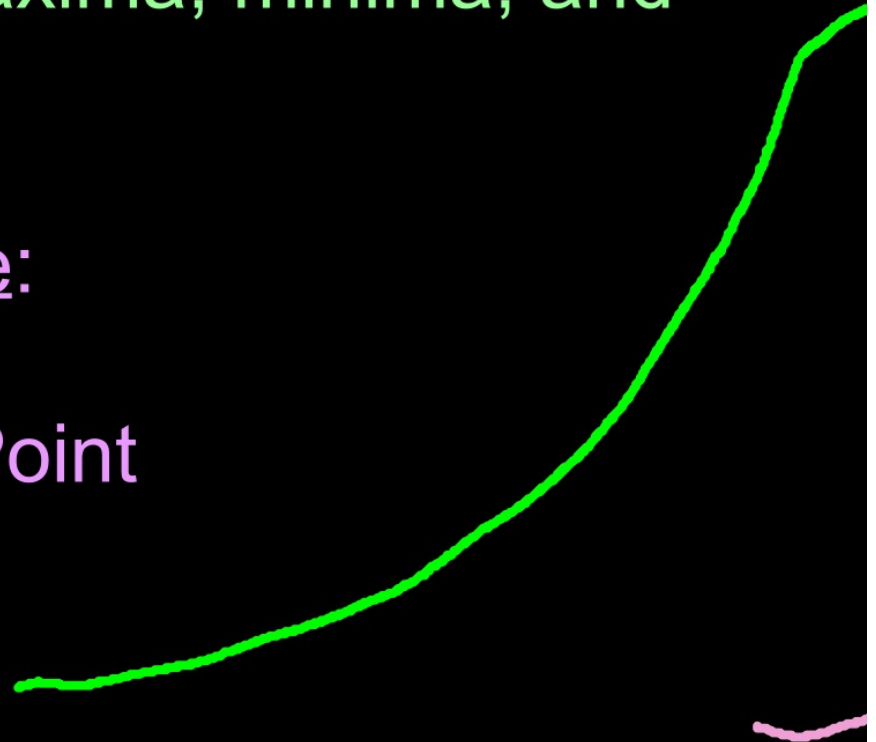


## Today's learning objective:

By the end of class, I will be able to identify how tangent lines behave at maxima, minima, and saddle points.

## Today's language objective:

Maxima, Minima, Saddle Point  
Differential Calculus  
Derivative  
Tangent  
Gradient



The output of the derivative  
is the gradient of the  
tangent.

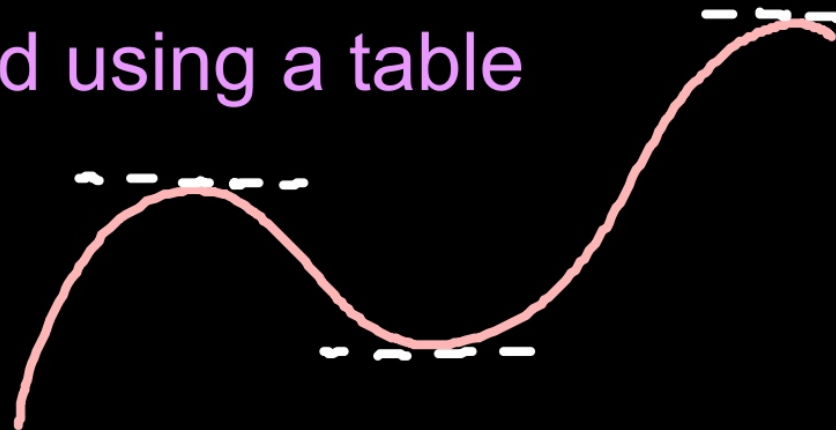
Explain to your partner what this means.

$$f'(x) = \underline{\quad}$$

Please graph on your board using a table  
(non-calculator question):

$$f(x) = x(x - 4)$$

$$x^2 - 4x$$



Where does the minimum or maximum appear to be?

Find  $f'(x)$   $2^{2-1} \rightarrow 2' \Rightarrow \boxed{2x-4}$   $f'(x) = 2x - 4 = 0$

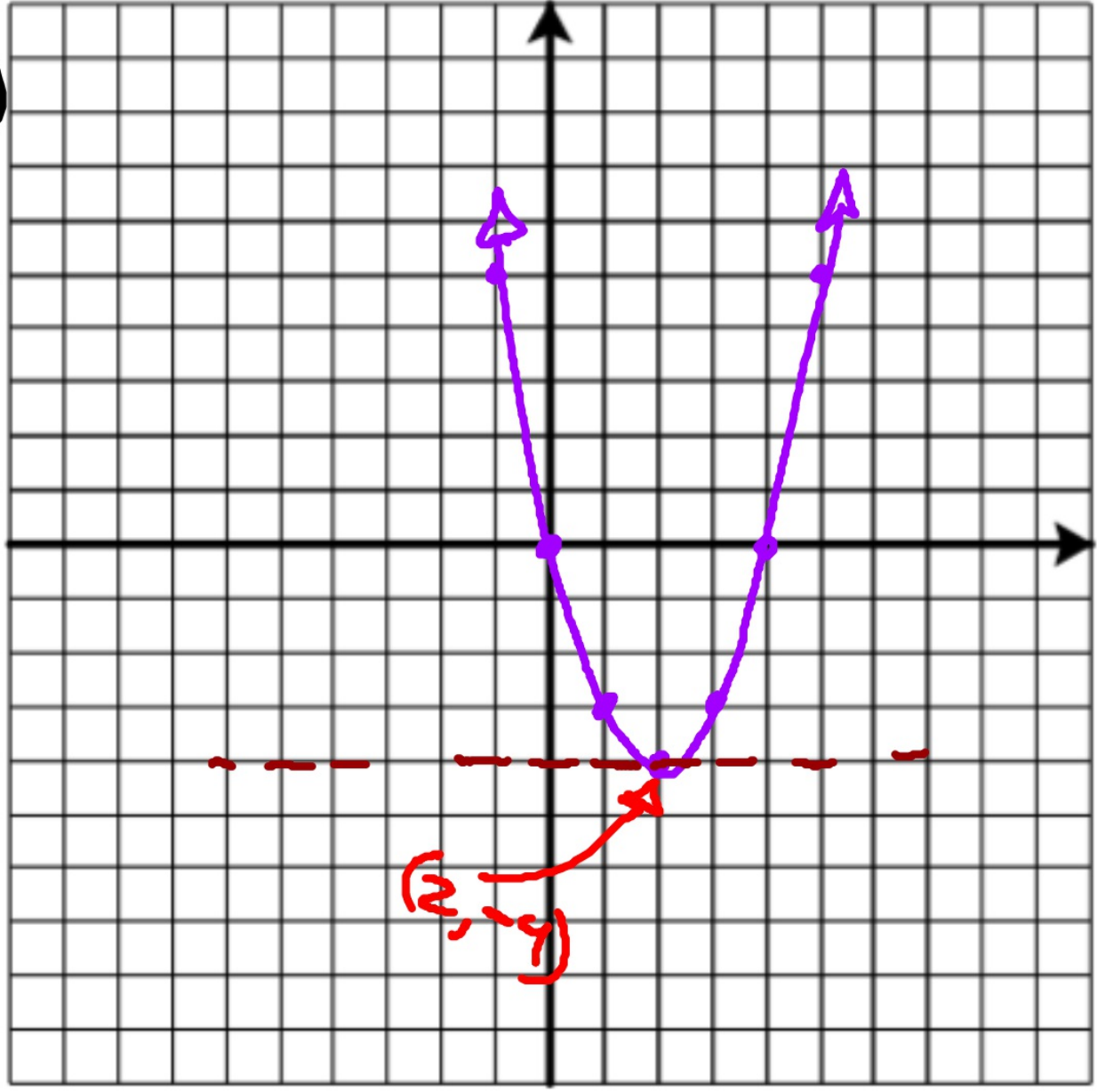
Solve  $f'(x) = 0$

(In other words, which value of "x" makes the derivative equal zero?)

$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

$$f(x) = x(x-4)$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 0 \\ 1 & -3 \\ 2 & -4 \\ 3 & -3 \\ 4 & 0 \end{array}$$



Find  $f'(x)$  for  $f(x) = \frac{x^3}{3} - x^2 - 3x + 2$

Non-CALC

Solve  $f'(x) = 0$  by factoring.

$$f'(x) = x^2 - 2x - 3$$

$$\frac{3x^2}{3} - 2x - 3$$

Without a graph, how would we know which x-value was the minimum or maximum?

Test the zeros by inputting numbers to the left and right of the potential min or max and make approximate tangent lines with those gradients.

Notice anything? :)



Extra space for calculations

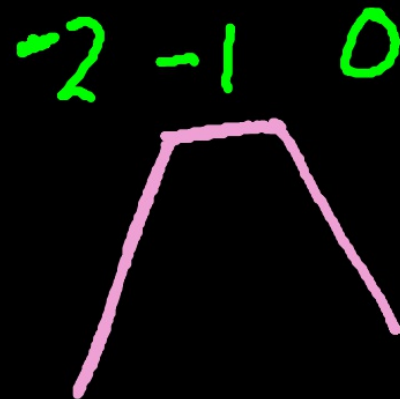
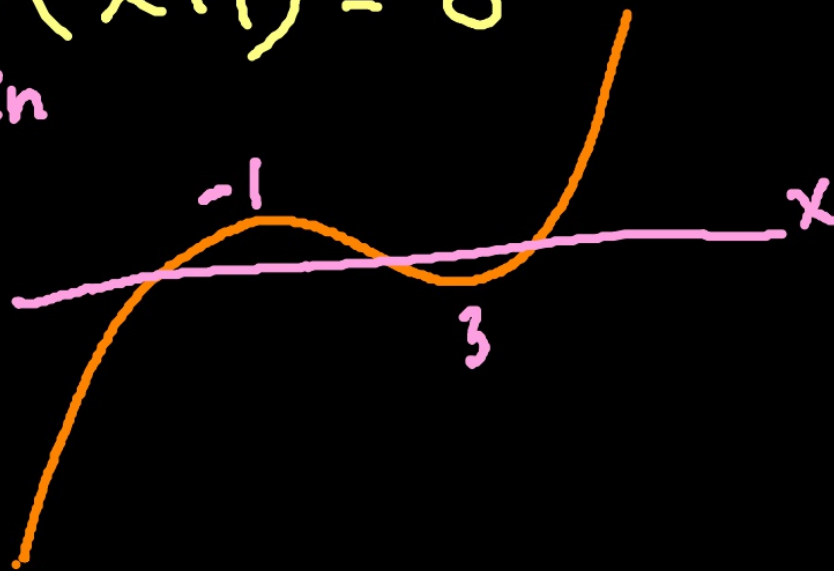
$$x^2 - 2x - 3$$

$$\begin{array}{r} (x \quad - \quad -3) \\ (x \quad + \quad +1) \\ \hline -3x \\ +x \\ \hline -2x \end{array}$$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ min}$$

$$x = -1 \text{ MAX}$$



Non. calc.

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} - 4$$

$$\text{find } f'(x) = \frac{3x^2}{3} - \frac{6x}{2} = x^2 - 3x$$

Solve  $f'(x) = 0$



$$x(x-3) = 0$$

min  $x=3$   
max  $x=0$

Test these solutions to see if they are minima or maxima.



$$f(x) = x^5 - 4x^3 + \frac{x}{4}$$

This is a calculator question.

$$\text{Find } f'(x) = 5x^4 - 12x^2 + \frac{1}{4}$$

Find the zeros using your calculator.

Now graph  $f(x)$  and find the min and max.

Notice anything? :)

## Saddle points

A saddle point is neither a min or max but nevertheless has a horizontal tangent line.

Graph  $f(x) = x^3$  in your calculator on a window of  $x\text{-min} = -3$ ,  $x\text{-max} = 3$ ,  $y\text{-min} = -10$ ,  $y\text{-max} = 10$ .

Where does the saddle point appear to be?

Find  $f'(x)$ .

Solve  $f'(x) = 0$  and test to see if it is a minimum or maximum with tangent lines.

Extra space for calculations.

