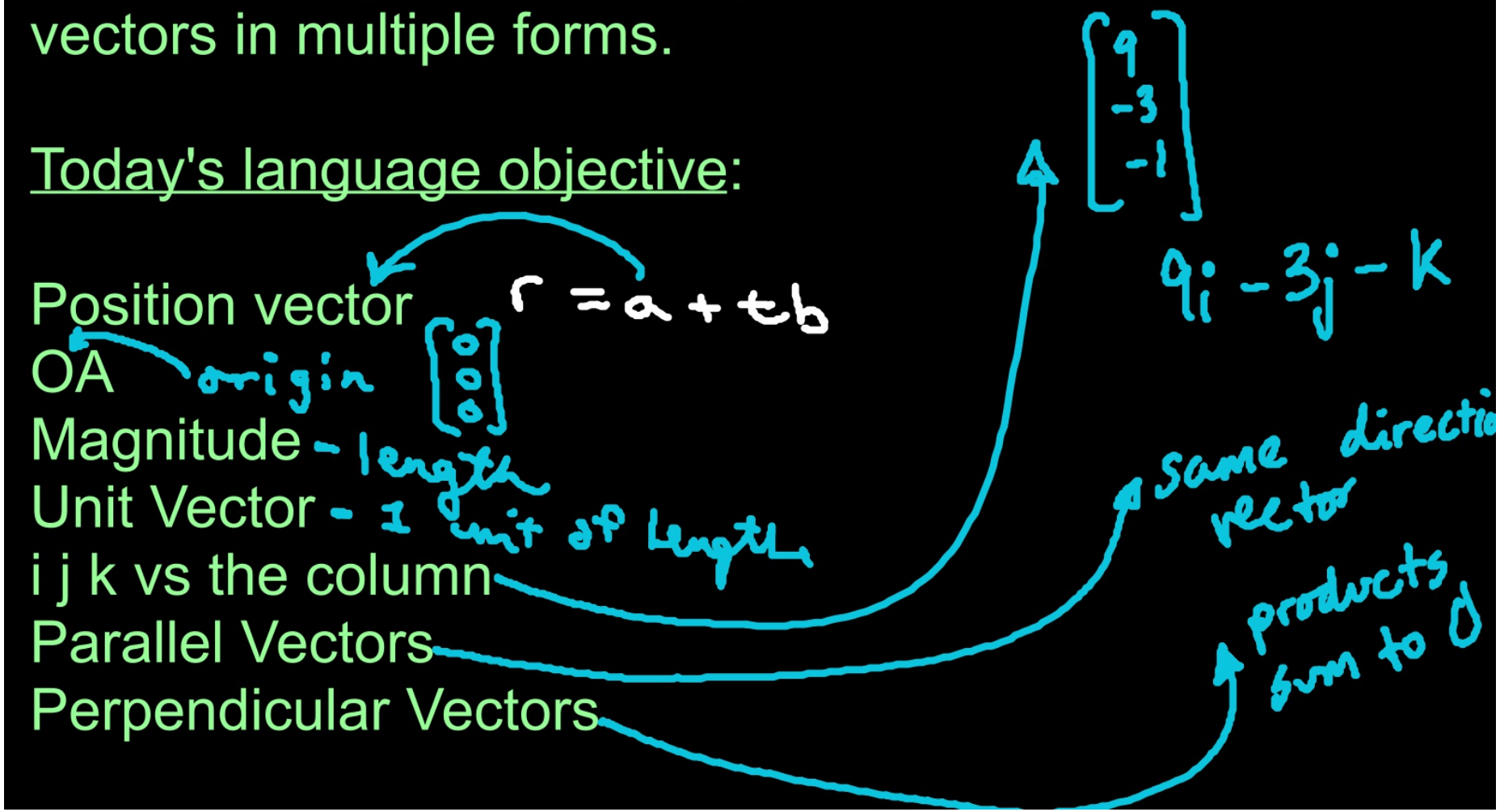
¿Quieren ustedes tirar piedras?

Veulent-ils jeter des pierres?

Today's learning objective:

By the end of class, I will be able to combine vectors, measure magnitude, identify the unit vector, and write vectors in multiple forms.

Today's language objective:



A line L passes through A$(1, -1, 2)$ and is parallel to the line $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

(a) Write down a vector equation for L in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

$L = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + t\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$

The line L passes through point P when $t = 2$.

(b) Find

(i) $\overrightarrow{OP}$;

$P \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(ii) $\left|\overrightarrow{OP}\right| = \sqrt{3^2 + 5^2 + (-2)^2} = \sqrt{38}$

6.) Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

(a) Find $\overrightarrow{BC}$.

C − B

$$\begin{bmatrix} -8 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$$

(b) Find a unit vector in the direction of $\overrightarrow{AB}$.

$$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2} = 7$$

$$\begin{bmatrix} 6/7 \\ -2/7 \\ 3/7 \end{bmatrix}$$

(c) Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$.

$6 \cdot -2 + -2 \cdot -3 + 3 \cdot 2 = 0$

14.) (a) Let $\boldsymbol{u} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}$. Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{w}$, find the value of p.

$6 - 3 - p = 0 \qquad p = 3$

(3

(b) Let $\boldsymbol{v} = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix}$. Given that $|\boldsymbol{v}| = \sqrt{42}$, find the possible values of q.

$\sqrt{42} = \sqrt{1^2 + q^2 + 5^2}$

$42 = 1 + q^2 + 25$

$vw = v_1w_1 + v_2w_2 + v_3w_3$

$16 = q^2$

$\pm 4 = q$

(3

(Total 6 mark

17.) Let $v = 3i + 4j + k$ and $w = i + 2j - 3k$. The vector $v + pw$ is perpendicular to w.

Find the value of p.

$$\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 3+p \\ 4+2p \\ 1-3p \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$$0 = 3 + p + 8 + 4p - 3 + 9p$$

$$14p + 8 = 0$$

$$p = -4/7$$

12.) The line L_1 is parallel to the z-axis. The point P has position vector $\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix}$ and lies on L_1.

(a) Write down the equation of L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. **(2)**

The line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$. The point A has position vector $\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix}$.

(b) Show that A lies on L_2. **(4)**

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22.) Consider the points A(5, 8), B(3, 5) and C(8, 6).

(a) Find

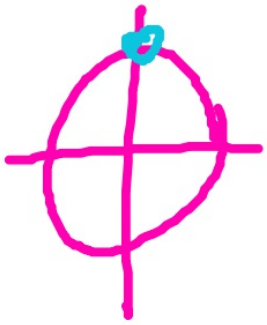
(i) $\overrightarrow{AB}$; $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

(ii) $\overrightarrow{AC}$. $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(b) (i) Find $\overrightarrow{AB} \bullet \overrightarrow{AC}$. $-6 + 6 = 0$

(ii) Find the sine of the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\sin 90° = 1$



26.) Consider the vectors $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = 4\boldsymbol{i} + \boldsymbol{j} - p\boldsymbol{k}$.

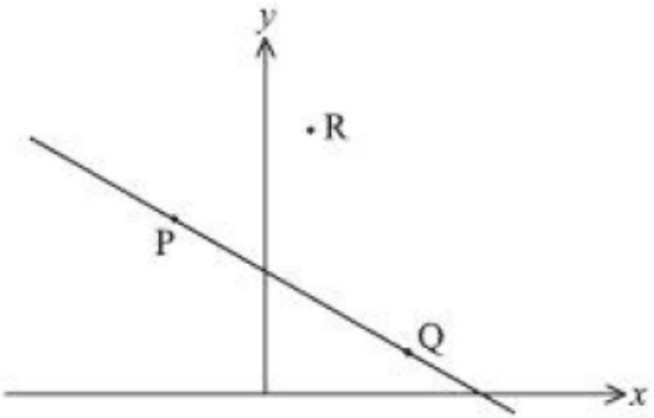
(a) Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ find the value of p.

(b) Given that $q|\mathbf{u}| = 14$, find the value of q.

27.) Points P and Q have position vectors $-5\boldsymbol{i} + 11\boldsymbol{j} - 8\boldsymbol{k}$ and $-4\boldsymbol{i} + 9\,\boldsymbol{j} - 5\boldsymbol{k}$ respectively, and both lie on a line L_1.

(a) (i) Find $\overrightarrow{PQ}$.

30.) The points P(−2, 4), Q (3, 1) and R (1, 6) are shown in the diagram below.



(a) Find the vector $\overrightarrow{PQ}$.

(b) Find a vector equation for the line through R parallel to the line (PQ).

(Total 6 marks)