NAME:____ **DATE: 09/13A-14B/2017**

ASSIGNMENT: Vector Intersection
DIRECTIONS: To find the intersection of two vectors, set them equal to one another. Then use elimination to eliminate one of the parameters, *t* or *s*.

After solving for a parameter, input it into the appropriate vector and add the separate dimensions to find your intersecting *x*, *y*, and *z*. See reverse for example.

5.) The line L_1 is represented by the vector equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point B(–8, –5, 25).

(a) Write down a vector equation for L_2 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(2)

A third line L_3 is perpendicular to L_1 and is represented by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q\begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

(b) Show that $k = -2$.

(5)

The lines L_1 and L_3 intersect at the point A.

(c) Find the coordinates of A.

(6)

The lines L_2 and L_3 intersect at point C where $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

(d) (i) Find $\overrightarrow{AB}$.

(ii) Hence, find $|\overrightarrow{AC}|$.

(5)
(Total 18 marks)

NAME:_______________________________________ **DATE: 09/13A-14B/2017**

13.) Two lines with equations $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the point P. Find the coordinates of P.

(Total 6 marks)

Step 1: For any value of *s* and *t*, let's find three true equations if the vectors intersect.

2 + 5s = 9 – 3t
3 – 3s = 2 + 5t
–1 + 2s = 2 – t

Step 2: Find the "lowest hanging fruit" for elimination of a variable. In other words, choose the two equations that yield the fastest path to eliminating "s" or "t." I think equations 2 and 3 are the best because the "t" looks easy to eliminate.

$$\begin{matrix} 3 - 3s = 2 + 5t \\ -1 + 2s = 2 - t \end{matrix} \longrightarrow \begin{matrix} 3 - 3s = 2 + 5t \\ 5(-1 + 2s) = 5(2 - t) \end{matrix} \longrightarrow \begin{matrix} 3 - 3s = 2 + 5t \\ -5 + 10s = 10 - 5t \end{matrix}$$

Step 3: Eliminate one variable and solve for the other.

3 – 3s = 2 + 5t
–5 + 10s = 10 – 5t

–2 + 7s = 12

7s = 14; s = 2

Step 4: Input your value of "s" into the appropriate vector (the one with the "s" parameter).

$$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$$

input 2 as "s"

$$\begin{matrix} 2 \\ 3 \\ -1 \end{matrix} \quad (2) \begin{matrix} 5 \\ -3 \\ 2 \end{matrix}$$

Step 5: Calculate your intersection position vector, Point P.

$$\begin{matrix} 2 + 10 \\ 3 - 6 \\ -1 + 4 \end{matrix} \longrightarrow \begin{matrix} 12 \\ -3 \\ 3 \end{matrix}$$