Let's go do some work.

$$W = \bar{F} \Delta s \cos\theta$$

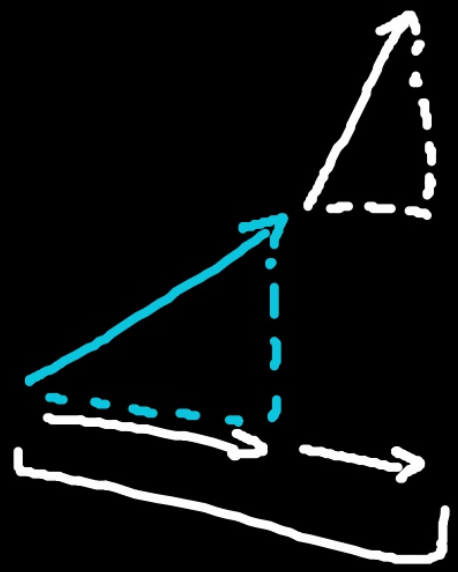
Today's learning objective:

By the end of class, I will be able to solve for the angle created by two intersecting vectors.

Today's language objective:

Magnitude
Scalar Multiplication
Cosine

$$v \cdot w = |v||w| \cos\theta$$



4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^{\,2} + v_2^{\,2} + v_3^{\,2}}$
4.2	Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$
		$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
	Angle between two vectors	$\cos\theta = \dfrac{\boldsymbol{v} \cdot \boldsymbol{w}}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$
4.3	Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

54.) Calculate the acute angle between the lines with equations

calc

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$4\cdot 1 + 3\cdot -1 = 1$$

$$\sqrt{3^2+4^2}\cdot\sqrt{1^2+(-1)^2} = \frac{1}{5\sqrt{2}} = \cos\theta$$

$\cos^{-1}(1/(5\sqrt{2}))$

$81.9°$

$\cos^{-1}$ $\cos^{-1}$

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
4.2	Scalar product •	$\boldsymbol{v}\cdot\boldsymbol{w} = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$ $\boldsymbol{v}\cdot\boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3$
	Angle between two vectors •	$\cos\theta = \dfrac{\boldsymbol{v}\cdot\boldsymbol{w}}{\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert}$
4.3	Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

38.) Find the cosine of the angle between the two vectors $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Non calc

$$\cos\theta = \frac{-2}{5\sqrt{5}}$$

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
4.2	Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$
		$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
	Angle between two vectors	$\cos\theta = \dfrac{\boldsymbol{v} \cdot \boldsymbol{w}}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$
4.3	Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

Find the cosine of the angle between the two vectors $3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$ and $4\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{k}$.

Non Calc

$$\cos\theta = \frac{-23}{50}$$

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^{\,2} + v_2^{\,2} + v_3^{\,2}}$
4.2	Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$
		$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
	Angle between two vectors	$\cos\theta = \dfrac{\boldsymbol{v} \cdot \boldsymbol{w}}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$

.) Two lines L_1 and L_2 are given by $\boldsymbol{r}_1 = \begin{pmatrix} 9 \\ 4 \\ -6 \end{pmatrix} + s\begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t\begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$.

(a) Let θ be the acute angle between L_1 and L_2. Show that $\cos\theta = \frac{52}{140}$.

(b) (i) P is the point on L_1 when $s = 1$. Find the position vector of P.

(ii) Show that P is also on L_2.

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
4.2	Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$
		$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
	Angle between two vectors	$\cos\theta = \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$

9.) Let $\boldsymbol{v} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\frac{\pi}{3}$.

Find the value of k.

~~MODE~~ = CALC

$\cos \pi/3 = \frac{1}{2}$

$\frac{1}{2} = \frac{2k+30}{7\sqrt{k^2+20}} - \frac{1}{2}$

$-1/2$

$0 = \frac{2k+30}{7\sqrt{k^2+20}} - \frac{1}{2}$

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
4.2	Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$
		$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
	Angle between two vectors	$\cos\theta = \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$
4.3	Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$\frac{0}{|v||w|} = 0$

$K = -4.23$

18.8

Y mi

Y ma

4.) Line L_1 passes through points A(1, –1, 4) and B(2, –2, 5).

(a) Find $\overrightarrow{AB}$.

(b) Find an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

Line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(c) Find the angle between L_1 and L_2.

(d) The lines L_1 and L_2 intersect at point C. Find the coordinates of C.