How is multiplication similar to division and vise versa?

inverse

$\times 4$

$\div \frac{1}{4}$

Today's learning objective:

By the end of class, I will be able to find the derivatives via the product and quotient rule.

Today's language objective:

I will use the term "Product rule" when I identify a function that requires the rule when differentiating.

6.2

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient rule

$y =$ [redacted]

$f(x) = \sin x$

$f'(x) = \cos x$

$$f(x) = \underbrace{4x^2}_{u} \cdot \underbrace{\sin 4x}_{v}$$

$$f'(x) = \underbrace{4x^2}_{u} \cdot \underbrace{\cos(4x) \cdot 4}_{v'} + \underbrace{\sin(4x)}_{v} \cdot \underbrace{8x}_{u'}$$

$$= 16x^2 \cos 4x + 8x \sin 4x$$

$$f(x) = \underbrace{24x^3}_{u}\,\underbrace{\cos 3x}_{v}$$

Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$f(x) = \cos x$

$f'(x) = -\sin x$

Show that $f'(x) = -72x^3 \sin 3x + 72x^2 \cos 3x$

$= 24x^3(-3\sin 3x) + \cos 3x \cdot 72x^2$

Derivative of x^n	$f(x)=x^n \Rightarrow f'(x)=nx^{n-1}$
Derivative of $\sin x$	$f(x)=\sin x \Rightarrow f'(x)=\cos x$
Derivative of $\cos x$	$f(x)=\cos x \Rightarrow f'(x)=-\sin x$
Derivative of $\tan x$	$f(x)=\tan x \Rightarrow f'(x)=\dfrac{1}{\cos^2 x}$
Derivative of e^x	$f(x)=e^x \Rightarrow f'(x)=e^x$
Derivative of $\ln x$	$f(x)=\ln x \Rightarrow f'(x)=\dfrac{1}{x}$
Chain rule	$y=g(u),\ u=f(x) \Rightarrow \dfrac{dy}{dx}=\dfrac{dy}{du}\times\dfrac{du}{dx}$
Product rule	$y=uv \Rightarrow \dfrac{dy}{dx}=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$
Quotient rule	$y=\dfrac{u}{v} \Rightarrow \dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$

What's another way to write the Product Rule?

$$f(x) \cdot g(x)$$

$$\Downarrow$$

$$f'(x) \cdot g'(x) = f(x) \cdot g'(x) + g(x) f'(x)$$

Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$f(x) = \sin x$

$f'(x) = \cos x$

$f(x) = (3x^2 - 4)\sin x$

find $f'(x)$

$f(x) = \underbrace{(3x^2-4)}_{u} \cdot \underbrace{\sin x}_{v}$

$(3x^2-4)\cdot(\cos x) + \sin(x)6x$

$\sin x$

$f(x) = 4x \ln x$

$\frac{4}{1}x \cdot \frac{1}{x} + \ln(x)4$

$\frac{4x}{x} + 4\ln x$

$4 + 4\ln x$

		$y = g(u),\ u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du}$
	Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
	Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$f(x) = \ln$

Challenge: find derivative of $f(x) = \frac{4x}{\ln x}$ (u / v)

$f'(x) =$

$\frac{4\ln x - 4}{(\ln x)^2} = \frac{\ln x \cdot 4 - 4x \cdot \frac{1}{x}}{(\ln x)^2}$

Differentiate:

$f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-3}$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

	Chain rule	$y = g(u),\ u = f(x)$ =
	Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx}$
	Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx}}$

$-2, 6, -24, 120$

$-3 \quad -4 \quad -5$

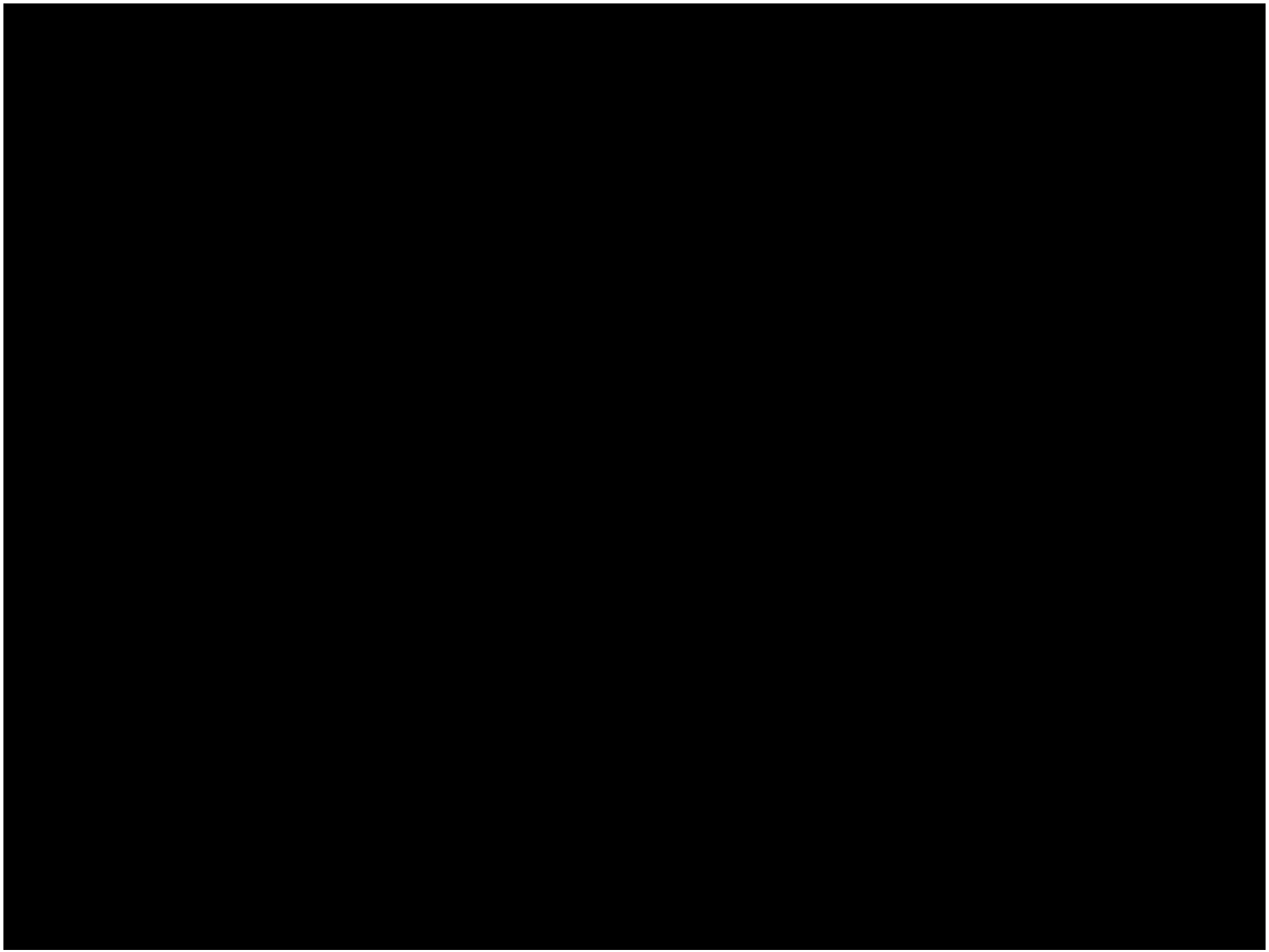
$6x^{-4} \quad -24x^{-5} \quad 120x^{-6}$

Challenge: find f''(x), f'''(x), and f''''(x).

$\cdot -101$

Now find the pattern.

$f^{100}(x) = x^{-102}$



Differentiate:

$h(x) = 4x^6 \tan x$

Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of $\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u),\ u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times$
Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Challenge: ln x cos 3x

Differentiate: Find $f'(\pi/2)$ for $f(x) = 3x^2 \sin x$

Challenge: Find $f''(\pi/2)$ for $f(x) = 3x^2 \sin x$

6.2	Derivative of x^n
	Derivative of $\sin x$

find $f'(x)$ for

$$f(x) = \tan x \, e^{4x}$$

Challenge

$$f(x) = \tan(2x) e^{4x} + 4x^2 \sin(x^2)$$

Derivative of $\sin x$	$f(x)=\sin x \Rightarrow f'(x)=\cos x$
Derivative of $\cos x$	$f(x)=\cos x \Rightarrow f'(x)=-\sin x$
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