28.) In this question, distance is in metres, time is in minutes.

Two model airplanes are each flying in a straight line.

At 13:00 the first model airplane is at the point (3, 2, 7). Its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$.

(a) Find the speed of the model airplane.

(2)

At 13:00 the second model airplane is at the point (– 5, 10, 23). After two minutes, it is at the point (3, 16, 39).

(b) Show that its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$.

(3)

(c) The airplanes meet at point Q.

(i) At what time do the airplanes meet?

(ii) Find the position of Q.

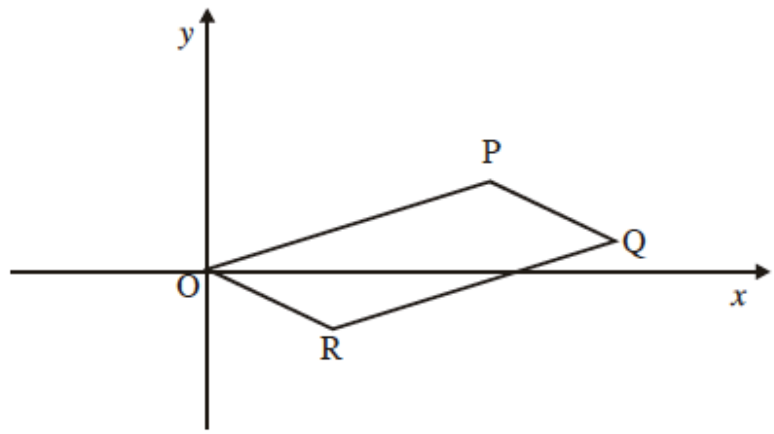
(6)

(d) Find the angle □ between the paths of the two airplanes.

(6)

(Total 17 marks)

48.) The diagram shows a parallelogram OPQR in which $\overrightarrow{OP} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$.



(a) Find the vector $\overrightarrow{OR}$.

(3)

(b) Use the scalar product of two vectors to show that $\cos O\hat{P}Q = -\frac{15}{\sqrt{754}}$.

(4)

(c) (i) Explain why $\cos P\hat{Q}R = -\cos O\hat{P}Q$.

(ii) Hence show that $\sin P\hat{Q}R = \frac{23}{\sqrt{754}}$.

(iii) Calculate the area of the parallelogram OPQR, giving your answer as an integer.

(7)

(Total 14 marks)

54.) Calculate the acute angle between the lines with equations

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

53.) Three of the coordinates of the parallelogram STUV are S(–2, –2), T(7, 7), U(5, 15).

(a) Find the vector $\overrightarrow{ST}$ and hence the coordinates of V.

(5)

(b) Find a vector equation of the line (UV) in the form ***r*** **=** ***p*** **+** □**d** where □ ∈ ℝ.

(2)

(c) Show that the point E with position vector $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$ is on the line (UV), and find the value of □ for this point.

(2)

The point W has position vector $\begin{pmatrix} a \\ 17 \end{pmatrix}$, $a \in \mathbb{R}$.

(d) (i) If $\left|\overrightarrow{EW}\right| = 2\sqrt{13}$, show that one value of a is –3 and find the other possible value of a.

(ii) For $a = -3$, calculate the angle between $\overrightarrow{EW}$ and $\overrightarrow{ET}$.

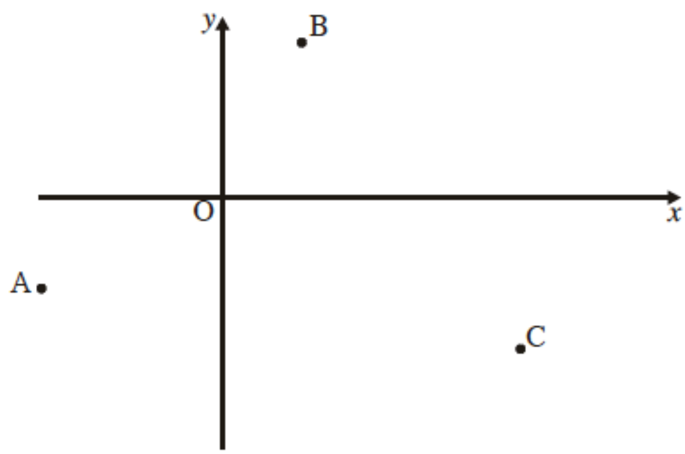
(10)
(Total 19 marks)

62.) A line passes through the point (4,–1) and its direction is perpendicular to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Find the equation of the line in the form $ax + by = p$, where a, b and p are integers to be determined.

39.) In this question the vector $\begin{pmatrix}1\\0\end{pmatrix}$ represents a displacement of 1 km east,

and the vector $\begin{pmatrix}0\\1\end{pmatrix}$ represents a displacement of 1 km north.

The diagram below shows the positions of towns A, B and C in relation to an airport O, which is at the point $(0, 0)$. An aircraft flies over the three towns at a constant speed of 250 km h^{-1}.



Town A is 600 km west and 200 km south of the airport.
Town B is 200 km east and 400 km north of the airport.
Town C is 1200 km east and 350 km south of the airport.

(a) (i) Find $\overrightarrow{AB}$.

(ii) Show that the vector of length one unit in the direction of $\overrightarrow{AB}$ is $\begin{pmatrix}0.8\\0.6\end{pmatrix}$.

(4)

An aircraft flies over town A at 12:00, heading towards town B at 250 km h^{-1}.

Let $\begin{pmatrix}p\\q\end{pmatrix}$ be the velocity vector of the aircraft. Let t be the number of hours in flight after 12:00.
The position of the aircraft can be given by the vector equation

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-600\\-200\end{pmatrix}+t\begin{pmatrix}p\\q\end{pmatrix}.$$

(b) (i) Show that the velocity vector is $\begin{pmatrix}200\\150\end{pmatrix}$.

(ii) Find the position of the aircraft at 13:00.

(iii) At what time is the aircraft flying over town B?

(6)

Over town B the aircraft changes direction so it now flies towards town C. It takes five hours to travel the 1250 km between B and C. Over town A the pilot noted that she had 17 000 litres of fuel left. The aircraft uses 1800 litres of fuel per hour when travelling at 250 km h^{-1}. When the fuel gets below 1000 litres a warning light comes on.

(c) How far from town C will the aircraft be when the warning light comes on?

(7)
(Total 17 marks)