Solve for x, that's the goal; .....

Today's learning objective:

By the end of class, I will be able to solve trigonometric equations by utilizing the unit circle and identities.

Today's language objective:

I will use the trigonometric identity term "double angle" and "Pythagorean identity" in my peer groups.

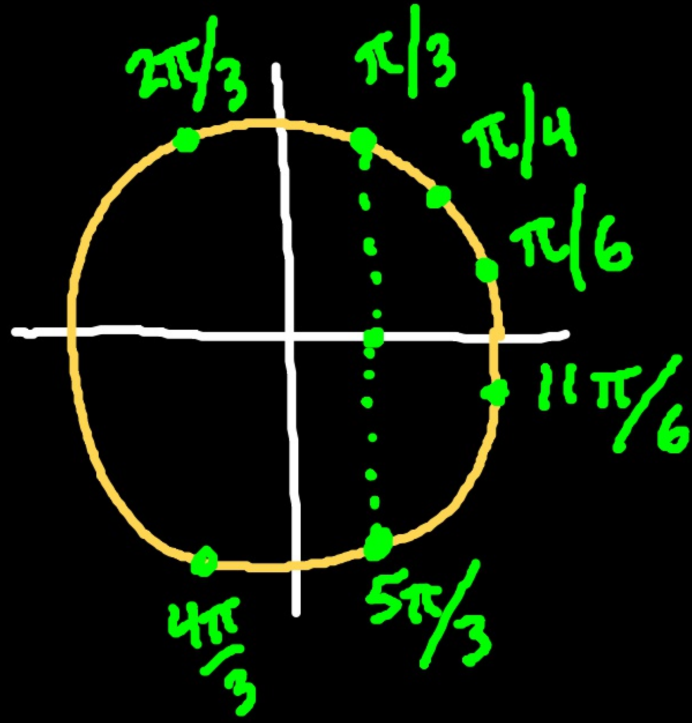
$$2\cos(x) - 1 = 0$$

N Calc

$$\frac{2\cos x}{2} = \frac{1}{2}$$

$$\cos x = \frac{1}{2}$$

$$x = \arccos\left(\frac{1}{2}\right)$$



$$x = \pi/3,\ 5\pi/3$$

3.1	Length of an arc	$l = \theta r$
	Area of a sector	$A = \frac{1}{2}\theta r^2$
3.2	Trigonometric identity	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
3.3	Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
	Double angle formulae	$\sin 2\theta = 2\sin\theta\cos\theta$
		$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
3.6	Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Area of a triangle	$A = \frac{1}{2}ab\sin C$

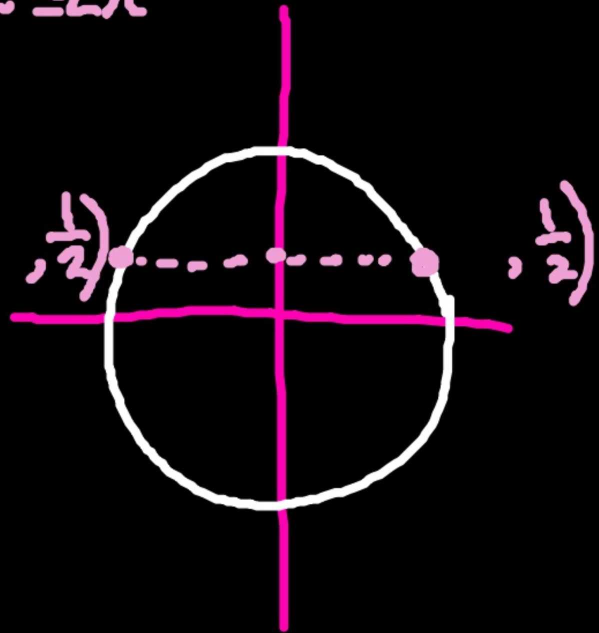
$2\sin x - 1 = 0$

$0 \le x \le 2\pi$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$\boxed{\pi/6,\ 5\pi/6}$

$30^\circ,\ 150^\circ$



3.2	Trigonometric $\boxed{\text{identity}}$	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
3.3	Pythagorean $\boxed{\text{identity}}$ Double angle formulae	$\cos^2\theta + \sin^2\theta = 1$ $\sin 2\theta = 2\sin\theta\cos\theta$ $\underline{\cos 2\theta} = \underline{\cos^2\theta - \sin^2\theta} = 2\cos^2\theta - 1 = \underline{1 - 2\sin^2\theta}$

$$\underbrace{2 + \cos \boxed{2x}}_{\text{substitute}} = 3\cos x$$

N-calc

$$2 + 2\cos^2 x - 1 = 3\cos x$$

$$2\cos^2 x + 1 = 3\cos x$$

$$2x^2 + 1 = 3x$$

$$2x^2 - 3x + 1 = 0$$

$\cos x = x$

$$2x^2 - 3x + 1 = 0$$

$(2x \quad - 1)$
$(x \quad - 1)$

$$(2x-1)(x-1) = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$$\boxed{\pi/3,\ 5\pi/3,\ 2\pi,\ 0}$$

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sin x = tan x

$$\sin x = \frac{\sin x}{\cos x}$$

$$\cos x \cdot \sin x = \sin x$$

$$\cos x = 1$$

$$x = 0, 2\pi$$

Have you ever seen "Identity?"

3.2	Trigonometric identity	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
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$\sin \underbrace{2x} = \cos x$ N-calc

$2\sin x \cos x = \cos x$

$2\sin x = 1$

$\pi/6, 5\pi/6$

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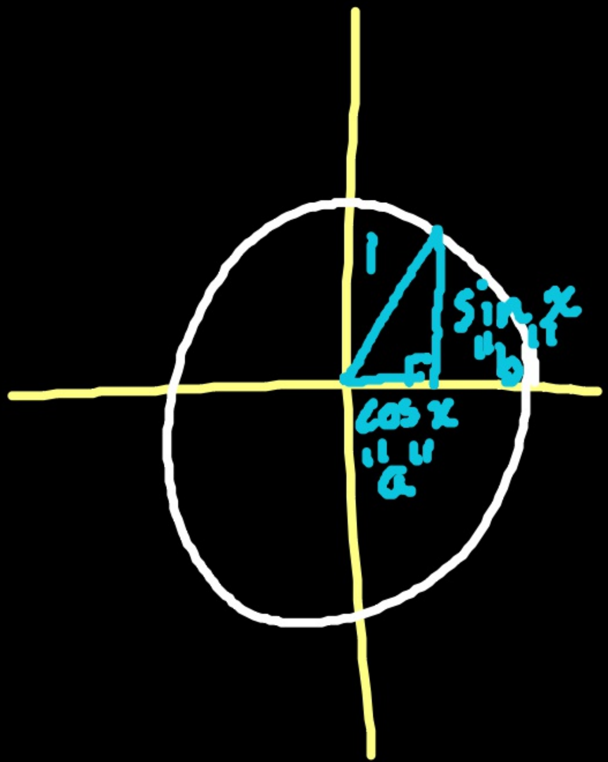
3.6	Cosine rule Sine rule Area of a triangle	$c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $A = \frac{1}{2}ab\sin C$

[redacted]

Why is the Pythagorean Identity true?



$\cos^2 x + \sin^2 x = 1^2$

Show that for $x = \pi$

$$\underbrace{\cos^2 x + \sin^2 x}_{1} = -\cos x$$

$$1 = -\cos x$$

$$-1 = \cos x$$

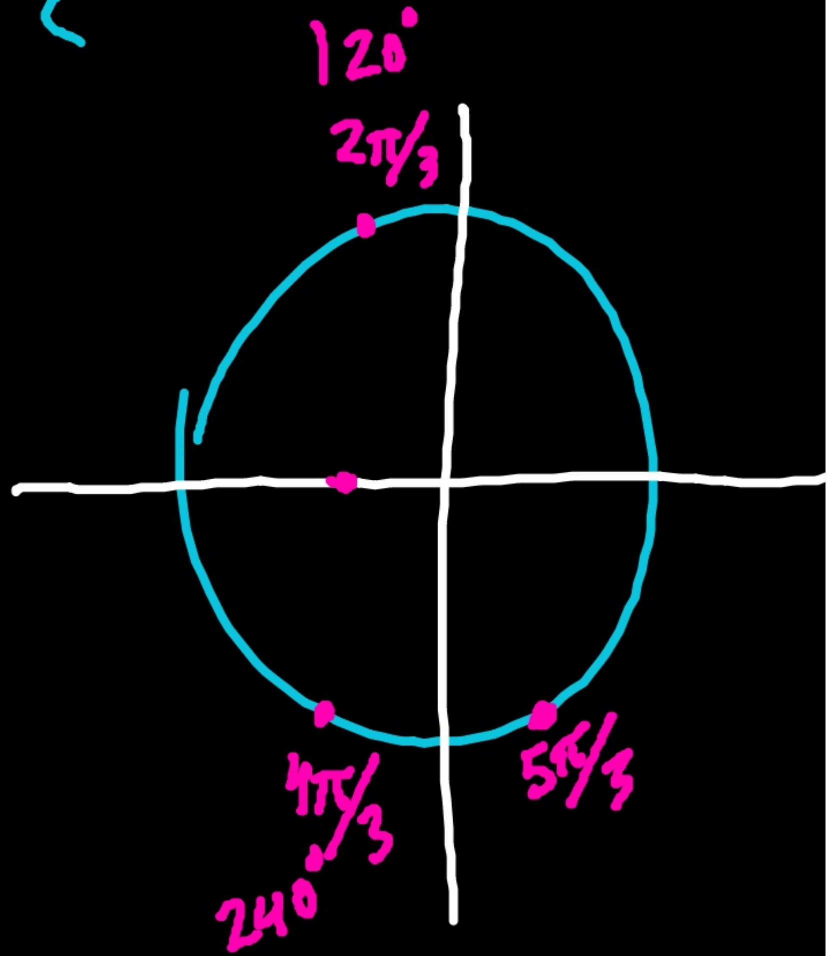
$$\boxed{x = \pi}$$

Show that for x = 240°

$$\left(\frac{\cos^2 x}{2} + \frac{\sin^2 x}{2}\right)^2 = \left(-\cos x\right)^2$$

$$1 = -2\cos x$$

$$-\frac{1}{2} = \cos x$$



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Show that for x = ~~π/2, 0~~

$$\sin 2x - \sin x = \sin^2 x + \cos^2 x$$

$2\underline{\sin x}\cos x - \underline{\sin x} = 1$

$\sin x(2\cos x - 1) = 1$

$\sin x = 1$

$2\cos x - 1 = 1$ $\quad\cos x = 1\quad$ $2\pi,\ 0$

Solve:

Show that "$7\cos x - 7\cos^2 x = 7\sin^2 x$" for $x = 0$

$+ 7\cos^2 x \qquad + 7\cos^2 x$

$$\frac{7\cos x}{7} = \frac{7\sin^2 x + 7\cos^2 x}{7}$$

$$\cos x = 1$$

$$x = 0, 2\pi$$

3.3	Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
	Double angle formulae	$\sin 2\theta = 2\sin\theta\cos\theta$
		$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Show that "$\cos 2x + 2\sin^2 x = \sin x$" is true for $x = \pi/2$

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Q-7. [Maximum marks 7] NC

(a) Given that $\cos A = \frac{1}{3}$ and $0 \leq A \leq \frac{\pi}{2}$, find $\cos 2A$.

$2\cos^2 A - 1 = \cos 2A$

$2\left(\frac{1}{3}\right)^2 - 1 = -7/9$

(b) Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \leq B \leq \pi$, find Sin2$B$.

$\sin 2B = 2\sin B\cos B$

$= 2\left(\frac{2}{3}\right)\cos B$

$= 4/3 \cos B = \frac{4}{3} \cdot \frac{\sqrt{5}}{3} = \boxed{\frac{4\sqrt{5}}{9}}$

$\cos^2 B + \left(\frac{2}{3}\right)^2 = 1$

$\cos^2 B = 5/9$

$\cos B = \sqrt{5}/3$

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(a) Let f be the function $f(\theta) = 2\cos 2\theta + 4\cos\theta + 3$, for $-0 \le \theta \le 2\pi$

Show that this function may be written as $(p\cos\theta + 1)^2$.

Write down the value of p.

$(2\cos x + 1)^2 = f(x)$

(4)

(b) Draw a rough sketch of the graph $f(\theta) - 0 \le \theta \le 2\pi$ **(3)**

(c) Consider the equation $f(\theta) = 0$, for $0 \le \theta \le 2\pi$.

Find all values of θ which satisfy this equation.

(2)

(d) Given that $f(\theta) = c$ is satisfied by only two values of θ, find the value of c.

(2)

(e) Consider the equation $f(\theta) = 1$, for $-2\pi \le \theta \le 2\pi$.

How many distinct values of $\cos\theta$ satisfy this equation? **(2)**

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[Maximum mark: 7]

Given that $\frac{\pi}{2} \le \theta \le \pi$ and that $\cos\theta = -\frac{12}{13}$, find

(a) $\sin\theta$; *[3 marks]*

(b) $\cos 2\theta$; *[3 marks]*

(c) $\sin(\theta + \pi)$. *[1 mark]*

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[Maximum mark: 6]

(a) Given that $2\sin^2\theta + \sin\theta - 1 = 0$, find the two values for $\sin\theta$. *[4 marks]*

(b) Given that $0^\circ \le \theta \le 360^\circ$ and that one solution for θ is 30°, find the other two possible values for θ. *[2 marks]*

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5.) Let $f(x) = \sin^3 x + \cos^3 x \tan x$, $\frac{\pi}{2} < x < \pi$.

(a) Show that $f(x) = \sin x$.

(b) Let $\sin x = \frac{2}{3}$. Show that $f(2x) = -\frac{4\sqrt{5}}{9}$.

(To

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