11/6

Solve for x, that's the goal; .....

Today's learning objective:

By the end of class, I will be able to solve trigonometric equations by utilizing the unit circle and identities.

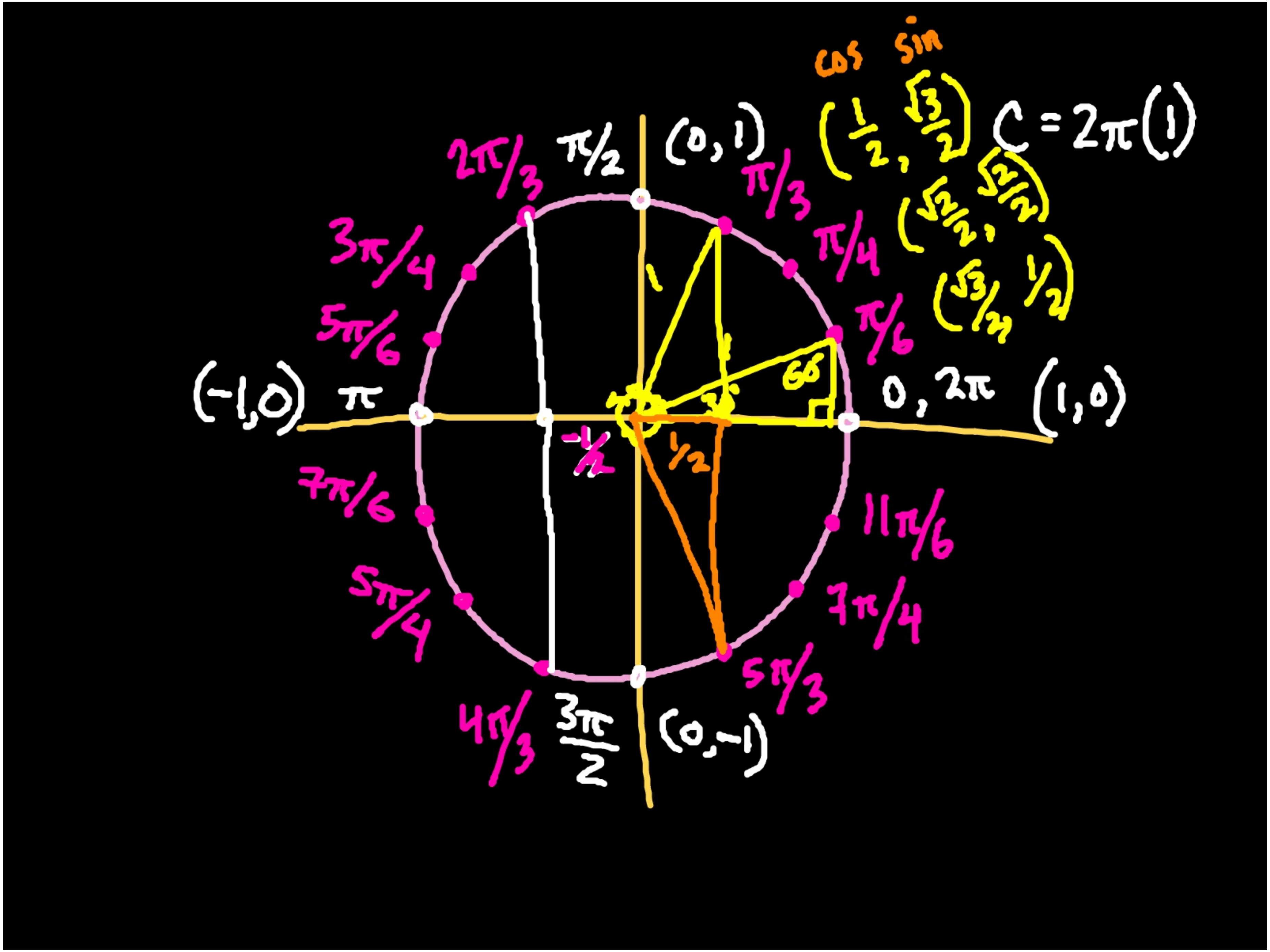
Today's language objective:

I will use the trigonometric identity term "double angle" and "Pythagorean identity" in my peer groups.

# Math IA

2 major assessments during Q4
- 1 solving trigonometric equations 11/7
- 1 Math IA (due on 12/7)

Math IA grades
*10 (70%)
*12 (80%)
*14 (90%)
*16 (100%)



cos sin
(1/2, √3/2)
C = 2π(1)
(√2/2, √2/2)
(√3/2, 1/2)
2π/3
π/2
(0, 1)
π/3
π/4
π/6
3π/4
5π/6
(−1, 0)
π
0, 2π
(1, 0)
60
1/2
−1/2
7π/6
11π/6
5π/4
7π/4
4π/3
3π/2
(0, −1)
5π/3

$$2\cos x - 1 = 0 \qquad 0 \le x \le 2\pi$$

$$\underline{2} \quad +1 \quad +1$$

$$\frac{\phantom{x}}{2}$$

$$\cos^{-1}(\cos x) = \frac{1}{2}$$

$$\cos^{-1}$$

$$x = \cos^{-1}\left(\frac{1}{2}\right) = \pi/3,\ 5\pi/3,\ 7\pi/3,\ -\pi/3$$

| | | |
|---|---|---|
| **3.1** | Length of an arc | $l = \theta r$ |
| | Area of a sector | $A = \frac{1}{2}\theta r^2$ |
| **3.2** | Trigonometric identity | $\tan\theta = \frac{\sin\theta}{\cos\theta}$ |
| **3.3** | Pythagorean identity | $\cos^2\theta + \sin^2\theta = 1$ |
| | Double angle formulae | $\sin 2\theta = 2\sin\theta\cos\theta$ |
| | | $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ |
| **3.6** | Cosine rule | $c^2 = a^2 + b^2 - 2ab\cos C$ ; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ |
| | Sine rule | $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ |
| | Area of a triangle | $A = \frac{1}{2}ab\sin C$ |

$2 \sin x - 1 = 0$

$-2\pi \le x \le 0$

$-\frac{7\pi}{6}, \ -\frac{11\pi}{6}$

| | | |
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$$2 + \underbrace{\cos \underbrace{2x}}\overset{=}{=} 3\cos x$$

1.5 = 1.5

$$2 + \underbrace{2\cos^2 x - 1} = 3\cos x$$

$$1 + 2\cos^2 x = 3\cos x \longrightarrow$$

$\cos x = x$

$2\cos x - 1 = 0$

$2\cos x - 1 = 0$

$\cos x - 1 = 0$

$$1 + 2x^2 = 3x$$

$$2x^2 - 3x + 1 = 0$$

$$\boxed{\pi/3,\ 5\pi/3,\ 2\pi,\ 0}$$

| | | |
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$\sin x = \tan x$ $\qquad 0 \leq x \leq 2\pi$

$\cos x = 1$

$x = 0,\ 2\pi$

# Have you ever seen "Identity?"

| | | |
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**$\sin 2x = \cos x$**

$\pi/6, 5\pi/6 = x$

$\sin x = \frac{1}{2}$

| | | |
|---|---|---|
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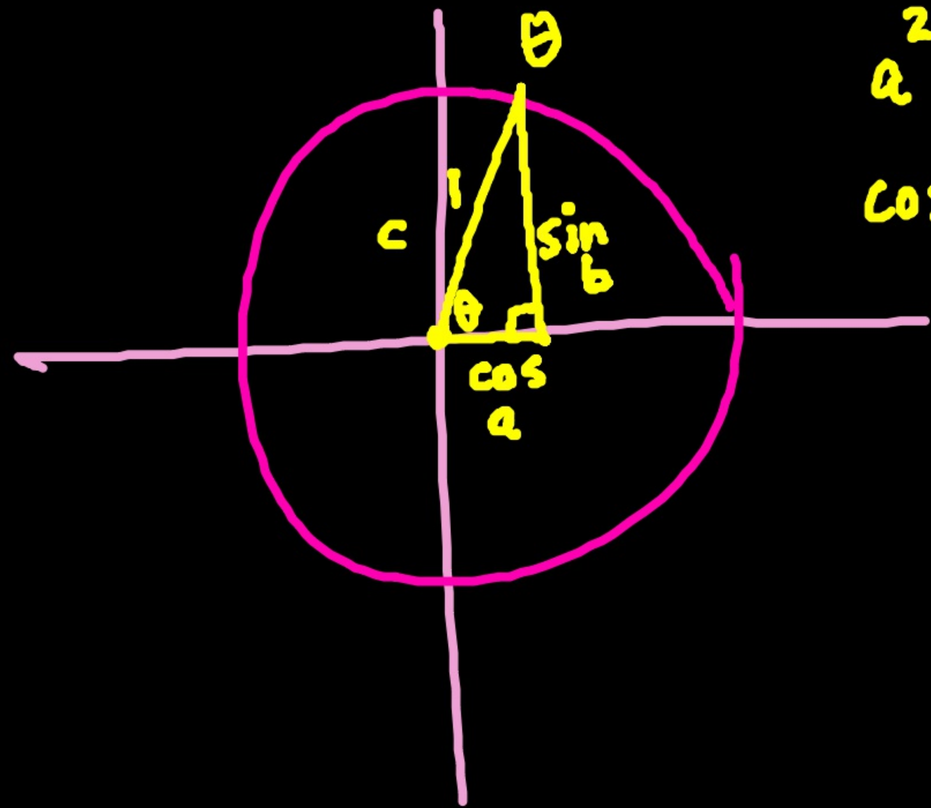
| **3.6** | Cosine rule<br><br>Sine rule<br><br>Area of a triangle | $c^2=a^2+b^2-2ab\cos C$ ; $\cos C=\frac{a^2+b^2-c^2}{2ab}$<br><br>$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$<br><br>$A=\frac{1}{2}ab\sin C$ |

[redacted]

# Why is the Pythagorean Identity true?




$$a^2 + b^2 = c^2$$

$$\cos^2\theta + \sin^2\theta = 1^2$$

Show that for $x = \pi$

$$\underbrace{\cos^2 x + \sin^2 x}_{1} = -\cos x$$

$$1 = -\cos x$$

$$-1 = \cos x$$

## Show that for x = 240°

$$\left[\frac{\cos^2 x}{2} + \frac{\sin^2 x}{2}\right]^2 = \left[-\cos x\right]^2$$

$$1 = -2\cos x$$

$$\frac{-1}{2} = \cos x$$

Show that $x = 0$ for

$$7\cos x - 7\cos^2 x = 7\sin^2 x$$

$$\cos x = 1$$

| | | |
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Show that "$\cos 2x + 2\sin^2 x = \sin x$" is true for $x = \pi/2$

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Q-7. [Maximum marks 7]

(a) Given that $\cos A = \frac{1}{3}$ and $0 \le A \le \frac{\pi}{2}$, find $\cos 2A$ $= 2\cos^2 A - 1 =$

$2\left(\frac{1}{3}\right)^2 - 1 = \boxed{\frac{-7}{9}}$

(b) Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\text{Sin}2B$ $= 2\sin B\cos B$

$2\left(\frac{2}{3}\right)\cos B$

$\boxed{\frac{4\sqrt{5}}{9}} = \frac{4}{3}\cancel{\cos B}\ \frac{\sqrt{5}}{3} =$

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$\left(\frac{2}{3}\right)^2$

$\frac{5}{9} = \cos^2\theta = \frac{\sqrt{5}}{3} = \cos\theta$

(a) Let $f$ be the function $f(\theta) = 2\cos 2\theta + 4\cos\theta + 3$, for $-0 \le \theta \le 2\pi$

Calc

Show that this function may be written as $(p\cos\theta + 1)^2$.

Write down the value of $p$.

**(4)**

(b) Draw a rough sketch of the graph $f(\theta)$ $-0 \le \theta \le 2\pi$ **(3)**

(c) Consider the equation $f(\theta) = 0$, for $0 \le \theta \le 2\pi$.

$2\cos x + 1 = 0$

$\cos x = \frac{-1}{2}$

Find all values of $\theta$ which satisfy this equation.

$2\pi/3$ ; $4\pi/3$

**(2)**

(d) Given that $f(\theta) = c$ is satisfied by only two values of $\theta$, find the value of $c$.

from

$y =$ some number

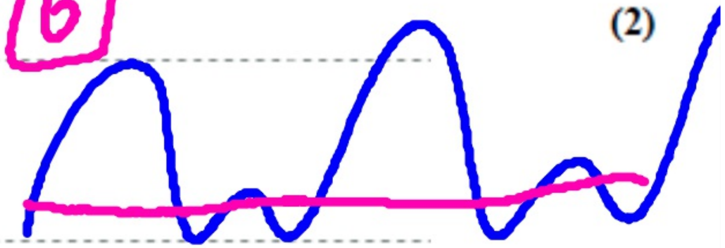
$c = 9$

$-2\pi < x \le 2\pi$

**(2)**

(e) Consider the equation $f(\theta) = 1$, for $-2\pi \le \theta \le 2\pi$.

How many distinct values of $\cos\theta$ satisfy this equation? $\boxed{6}$ **(2)**



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*[Maximum mark: 7]*

Non Calc

$\frac{5}{13} = \sqrt{\frac{25}{169}} = \frac{169}{169} - \frac{144}{169} = \sqrt{\sin^2\theta}$

Given that $\frac{\pi}{2} \le \theta \le \pi$ and that $\cos\theta = -\frac{12}{13}$, find

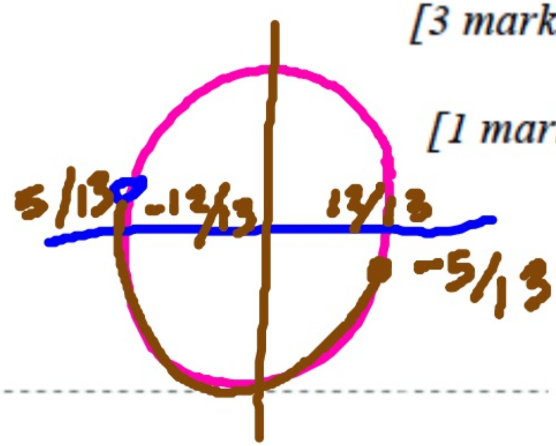
$\frac{144}{169} + \sin^2\theta = 1$

(a) $\sin\theta$; $\cos^2\theta + \sin^2\theta = 1$ *[3 marks]*

$5/13$

(b) $\cos 2\theta$; $\left(-12/13\right)^2$ $119/169$ *[3 marks]*

(c) $\sin(\theta + \pi)$. $-5/13$ *[1 mark]*

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$\frac{144}{169} - \frac{25}{169}$

*[Maximum mark: 6]*

(a) Given that $2\sin^2\theta + \sin\theta - 1 = 0$, find the two values for $\sin\theta$. *[4 marks]*

(b) Given that $0^\circ \le \theta \le 360^\circ$ and that one solution for $\theta$ is $30^\circ$, find the other two possible values for $\theta$. *[2 marks]*

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5.) Let $f(x) = \sin^3 x + \cos^3 x \tan x$, $\frac{\pi}{2} < x < \pi$.

(a) Show that $f(x) = \sin x$.

(b) Let $\sin x = \frac{2}{3}$. Show that $f(2x) = -\frac{4\sqrt{5}}{9}$.

(To

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