1.) (a) $v = 1$ A1 N1 1

(b) (i) $\frac{\mathrm{d}}{\mathrm{d}t}(2t) = 2$ A1

$\frac{\mathrm{d}}{\mathrm{d}t}(\cos 2t) = -2\sin 2t$ A1A1

Note: *Award A1 for coefficient 2 and A1 for –sin 2t.*

evidence of considering acceleration = 0 (M1)

e.g. $\frac{\mathrm{d}v}{\mathrm{d}t} = 0, 2 - 2\sin 2t = 0$

correct manipulation A1

e.g. $\sin 2k = 1,\ \sin 2t = 1$

$2k = \frac{\pi}{2}\left(\text{accept } 2t = \frac{\pi}{2}\right)$ A1

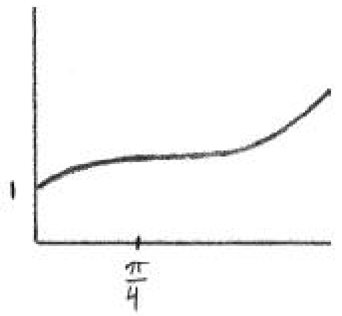
$k = \frac{\pi}{4}$ AG N0

(ii) attempt to substitute $t = \frac{\pi}{4}$ into v (M1)

e.g. $2\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right)$

$v = \frac{®}{2}$ A1 N28

(c)



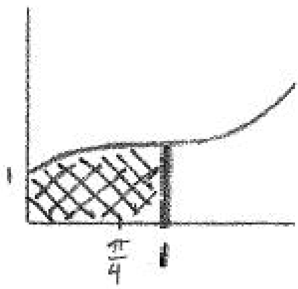
A1A1A2 N44

Notes: *Award A1 for y-intercept at (0, 1), A1 for curve having zero gradient at* $t = \frac{\pi}{4}$*, A2 for shape that is concave down to the left of* $\frac{\pi}{4}$ ***and*** *concave up to the right of* $\frac{\pi}{4}$*. If a correct curve is drawn without indicating* $t = \frac{\pi}{4}$*, do not award the second A1 for the zero gradient, but award the final A2 if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.*

(d) (i) correct expression A2

e.g. $\int_0^1 (2t + \cos 2t)\mathrm{d}t, \left[t^2 + \frac{\sin 2t}{2}\right]_0^1, 1 + \frac{\sin 2}{2}, \int_0^1 v\mathrm{d}t$

(ii)



A1 3

Note: *The line at t = 1 needs to be clearly after* $t = \frac{\pi}{4}$.

[16]

2.) (a) $f(1) = 2$(A1)

$f'(x) = 4x$ A1

evidence of finding the gradient of f at $x = 1$ M1

e.g. substituting $x = 1$ into $f'(x)$

finding gradient of f at $x = 1$ A1

e.g. $f'(1) = 4$

evidence of finding equation of the line M1

e.g. $y - 2 = 4(x - 1)$, $2 = 4(1) + b$

$y = 4x - 2$ AG N05

(b) appropriate approach (M1)

e.g. $4x - 2 = 0$

$x = \frac{1}{2}$ A1 N22

(c) (i) bottom limit $x = 0$ (seen anywhere) (A1)

approach involving subtraction of integrals/areas (M1)

e.g. $\int f(x)$ – area of triangle, $\int f - \int l$

correct expression A2 N4

e.g. $\int_0^1 2x^2 \mathrm{d}x - \int_{0.5}^1 (4x-2)\mathrm{d}x, \int_0^1 f(x)\mathrm{d}x - \frac{1}{2}, \int_0^{0.5} 2x^2 \mathrm{d}x + \int_{0.5}^1 f(x) - (4x-2)\mathrm{d}x$

(ii) **METHOD 1 (using only integrals)**

correct integration (A1)(A1)(A1)

$\int 2x^2 \mathrm{d}x = \frac{2x^3}{3}, \int (4x-2)\mathrm{d}x = 2x^2 - 2x$

substitution of limits (M1)

e.g. $\frac{1}{12} + \frac{2}{3} - 2 + 2 - \left(\frac{1}{12} - \frac{1}{2} + 1\right)$

area = $\frac{1}{6}$ A1 N4

METHOD 2 (using integral and triangle)

area of triangle= $\frac{1}{2}$ (A1)

correct integration (A1)

$\int 2x^2 \mathrm{d}x = \frac{2x^3}{3}$

substitution of limits (M1)

e.g. $\frac{2}{3}(1)^3 - \frac{2}{3}(0)^3, \frac{2}{3} - 0$

correct simplification (A1)

e.g. $\frac{2}{3} - \frac{1}{2}$

area = $\frac{1}{6}$ A1 N49

[16]

3.) evidence of finding intersection points (M1)

e.g. $f(x) = g(x)$, $\cos x^2 = \mathrm{e}^x$, sketch showing intersection

$x = -1.11$, $x = 0$ (may be seen as limits in the integral) A1A1

evidence of approach involving integration and subtraction (in any order)(M1)

e.g. $\int_{-1.11}^{0} \cos x^2 - \mathrm{e}^x, \int (\cos x^2 - \mathrm{e}^x)\,\mathrm{d}x, \int g - f$

area = 0.282 A2 N3

[6]

4.)

METHOD 1

evidence of antidifferentiation (M1)

e.g. $\int(10\mathrm{e}^{2x} - 5)\mathrm{d}x$

$y = 5\mathrm{e}^{2x} - 5x + C$ A2A1

Note: *Award A2 for $5\mathrm{e}^{2x}$, A1 for $-5x$. If "C" is omitted, award no further marks.*

substituting (0, 8) (M1)

e.g. $8 = 5 + C$

$C = 3$ ($y = 5\mathrm{e}^{2x} - 5x + 3$) (A1)

substituting $x = 1$ (M1)

$y = 34.9$ ($5\mathrm{e}^2 - 2$) A1 N48

METHOD 2

evidence of definite integral function expression (M2) *e.g.* $\int_a^x f'(t)\mathrm{d}t = f(x) - f(a), \int_0^x (10\mathrm{e}^{2x} - 5)$

initial condition in definite integral function expression (A2)

e.g. $\int_0^x (10\mathrm{e}^{2t} - 5)\mathrm{d}t = y - 8, \int_0^x (10\mathrm{e}^{2x} - 5)\mathrm{d}x + 8$

correct definite integral expression for y when x =1 (A2)

e.g. $\int_0^1 (10\mathrm{e}^{2x} - 5)\mathrm{d}x + 8$

y = 34.9 ($5\mathrm{e}^2 - 2$) A2 N48

5.)

attempt to set up integral expression M1

e.g. ® $\int \sqrt{16-4x^2}^{\,2}\mathrm{d}x, 2\pi\int_0^2 (16-4x^2), \int \sqrt{16-4x^2}^{\,2}\mathrm{d}x$

$\int 16\mathrm{d}x = 16x, \int 4x^2\mathrm{d}x = \frac{4x^3}{3}$ (seen anywhere) A1A1

evidence of substituting limits into the integrand (M1)

e.g. $\left(32 - \frac{32}{3}\right) - \left(-32 + \frac{32}{3}\right), 64 - \frac{64}{3}$

volume = $\frac{128\pi}{3}$ A2 N3

[6]

6.) (a) substituting into the second derivative M1

e.g. $3 \times \left(-\frac{4}{3}\right) - 1$

$f''\left(-\frac{4}{3}\right) = -5$ A1

since the second derivative is negative, B is a maximum R1 N0

(b) setting $f'(x)$ equal to zero (M1)

evidence of substituting $x = 2$ $\left(\text{or } x = -\frac{4}{3}\right)$ (M1)

e.g. $f'(2)$

correct substitution A1

e.g. $\frac{3}{2}(2)^2 - 2 + p, \frac{3}{2}\left(-\frac{4}{3}\right)^2 - \left(-\frac{4}{3}\right) + p$

correct simplification

e.g. $6 - 2 + p = 0$, $\frac{8}{3} + \frac{4}{3} + p = 0$, $4 + p = 0$ A1

$p = -4$ AGN0

(c) evidence of integration (M1)

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + c$ A1A1A1

substituting (2, 4) or $\left(-\frac{4}{3}, \frac{358}{27}\right)$ into **their** expression (M1)

correct equation A1

e.g. $\frac{1}{2}\times 2^3 - \frac{1}{2}\times 2^2 - 4\times 2 + c = 4, \frac{1}{2}\times 8 - \frac{1}{2}\times 4 - 4\times 2 + c = 4, 4 - 2 - 8 + c = 4$

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 10$ A1N4

[14]

(7)
(Total 14 marks)

7.) (a) (i) $\sin x = 0$ A1
$x = 0, x = \pi$ A1A1 N2

(ii) $\sin x = -1$ A1

$x = \frac{3\pi}{2}$ A1N1

(b) $\frac{3\pi}{2}$ A1N1

(c) evidence of using anti-differentiation (M1)

e.g. $\int_0^{\frac{3\pi}{2}} (6 + 6\sin x)\mathrm{d}x$

correct integral $6x - 6\cos x$ (seen anywhere) A1A1

correct substitution (A1)

e.g. $6\left(\frac{3\pi}{2}\right) - 6\cos\left(\frac{3\pi}{2}\right) - (-6\cos 0)$, $9\pi - 0 + 6$

$k = 9\pi + 6$ A1A1N3

(d) translation of $\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$ A1A1N2

(e) recognizing that the area under *g* is the same as the shaded region in *f* (M1)

$p = \frac{\pi}{2}, p = 0$ A1A1N3

[17]

8.) evidence of integrating the acceleration function (M1)

e.g. $\int\left(\frac{1}{t} + 3\sin 2t\right)\mathrm{d}t$

correct expression $\ln t - \frac{3}{2}\cos 2t + c$ A1A1

evidence of substituting (1, 0) (M1)

e.g. $0 = \ln 1 - \frac{3}{2}\cos 2 + c$

$c = -0.624 \left(= \frac{3}{2}\cos 2 - \ln 1 \text{ or } \frac{3}{2}\cos 2\right)$ (A1)

$v = \ln t - \frac{3}{2}\cos 2t - 0.624 \left(= \ln t - \frac{3}{2}\cos 2t + \frac{3}{2}\cos 2 \text{ or } \ln t - \frac{3}{2}\cos 2t + \frac{3}{2}\cos 2 - \ln 1\right)$ (A1)

$v(5) = 2.24$ (accept the exact answer $\ln 5 - 1.5\cos 10 + 1.5\cos 2$) A1 N3

[7]

9.) (a) substituting (0, 13) into function M1

e.g. $13 = Ae^0 + 3$

$13 = A + 3$ A1

$A = 10$ AG N0

(b) substituting into $f(15) = 3.49$ A1

e.g. $3.49 = 10e^{15k} + 3$, $0.049 = e^{15k}$

evidence of solving equation (M1)

e.g. sketch, using ln

$k = -0.201 \left(\text{accept } \frac{\ln 0.049}{15}\right)$ A1N2

(c) (i) $f(x) = 10e^{-0.201x} + 3$

$f'(x) = 10e^{-0.201x} \times -0.201$ $(= -2.01e^{-0.201x})$A1A1A1 N3

Note: *Award A1 for* $10e^{-0.201x}$, *A1 for* $\times -0.201$, *A1 for the derivative of 3 is zero.*

(ii) valid reason with reference to derivative R1N1

e.g. $f'(x) < 0$, derivative always negative

(iii) $y = 3$ A1N1

(d) finding limits 3.8953…, 8.6940… (seen anywhere) A1A1

evidence of integrating and subtracting functions (M1)

correct expression A1

e.g. $\int_{3.90}^{8.69} g(x) - f(x)\mathrm{d}x$, $\int_{3.90}^{8.69} [(-x^2 + 12x - 24) - (10e^{-0.201x} + 3)]\mathrm{d}x$

area = 19.5 A2N4

10.) (a) 2.31 A1 N1

(b) (i) 1.02 A1 N1

(ii) 2.59 A1N1

(c) $\int_p^q f(x)\mathrm{d}x = 9.96$ A1N1

split into two regions, make the area below the *x*-axis positive R1R1N2

[6]

11.) **evidence of integration**

***e.g.* $f(x) = \int \sin(2x - 3)\mathrm{d}x$ (M1)**

= $-\frac{1}{2}\cos(2x-3)+C$ **A1A1**

substituting initial condition into **their** expression (even if C is missing) M1

e.g. $4 = -\frac{1}{2}\cos 0 + C$

$C = 4.5$ (A1)

$f(x) = -\frac{1}{2}\cos(2x-3)+4.5$ A1 N5

[6]

(Total 6 marks)

12.) (a) (i) substitute into gradient = $\frac{y_1-y_2}{x_1-x_2}$ (M1)

e.g. $\frac{f(a)-0}{a-\frac{2}{3}}$

substituting $f(a) = a^3$

e.g. $\frac{a^3-0}{a-\frac{2}{3}}$ A1

gradient = $\frac{a^3}{a-\frac{2}{3}}$ AGN0

(ii) correct answer A1N1

e.g. $3a^2$, $f'(a) = 3$, $f'(a) = \frac{a^3}{a-\frac{2}{3}}$

(iii) **METHOD 1**

evidence of approach (M1)

e.g. $f'(a)$ = gradient, $3a^2 = \frac{a^3}{a-\frac{2}{3}}$

simplify A1

e.g. $3a^2\left(a-\frac{2}{3}\right) = a^3$

rearrange A1

e.g. $3a^3 - 2a^2 = a^3$

evidence of solving A1

e.g. $2a^3 - 2a^2 = 2a^2(a-1) = 0$

$a = 1$ AGN0

METHOD 2

gradient RQ = $\dfrac{-8}{-2-\frac{2}{3}}$ A1

simplify A1

e.g. $\dfrac{-8}{-\frac{8}{3}}$, 3

evidence of approach (M1)

e.g. $f'(a)$ = gradient, $3a^2 = \dfrac{-8}{-2-\frac{2}{3}}, \dfrac{a^3}{a-\frac{2}{3}} = 3$

simplify A1

e.g. $3a^2 = 3$, $a^2 = 1$

$a = 1$ AGN0

(b) approach to find area of T involving subtraction and integrals (M1)

e.g. $\int f-(3x-2)\mathrm{d}x, \int_{-2}^{k}(3x-2)-\int_{-2}^{k}x^3, \int(x^3-3x+2)$

correct integration with correct signs A1A1A1

e.g. $\frac{1}{4}x^4-\frac{3}{2}x^2+2x, \frac{3}{2}x^2-2x-\frac{1}{4}x^4$

correct limits –2 and k (seen anywhere) A1

e.g. $\int_{-2}^{k}(x^3-3x+2)\mathrm{d}x, \left[\frac{1}{4}x^4-\frac{3}{2}x^2+2x\right]_{-2}^{k}$

attempt to substitute k and –2 (M1)

correct substitution into **their** integral if 2 or more terms A1

e.g. $\left(\frac{1}{4}k^4-\frac{3}{2}k^2+2k\right)-(4-6-4)$

setting **their** integral expression equal to $2k + 4$ (seen anywhere) (M1)

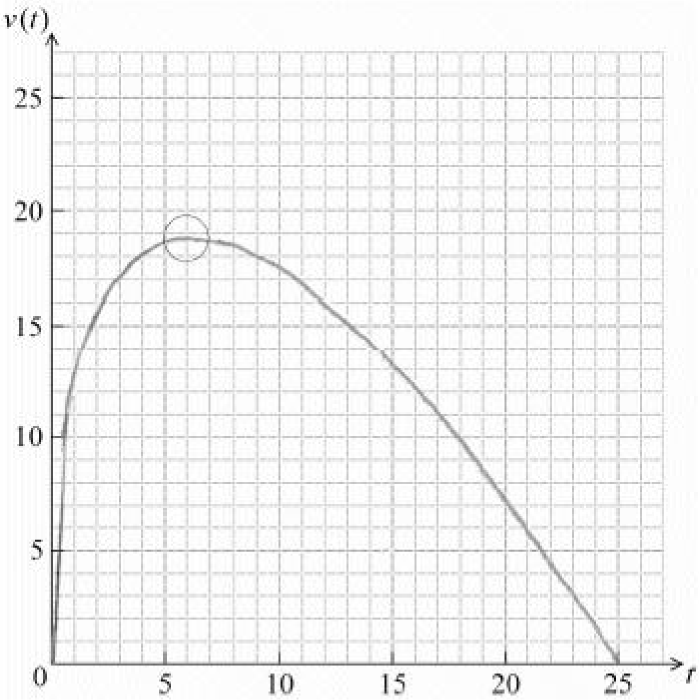
simplifying A1

e.g. $\frac{1}{4}k^4-\frac{3}{2}k^2+2=0$

$k^4 - 6k^2 + 8 = 0$ AGN0

[16]

13.) (a)



A1A1A1 N3

Note: *Award A1 for approximately correct shape, A1 for right endpoint at (25, 0) and A1 for maximum point in circle.*

(b) (i) recognizing that d is the area under the curve (M1)

e.g. $\int v(t)$

correct expression in terms of t, with correct limits A2N3

e.g. $d = \int_0^9 (15\sqrt{t} - 3t)\mathrm{d}t,\ d = \int_0^9 v\mathrm{d}t$

(ii) $d = 148.5$ (m) (accept 149 to 3 sf) A1N1

[7]

14.) (a) evidence of valid approach (M1)

e.g. $f(x) = 0$, graph

$a = -1.73$, $b = 1.73$ $(a = -\sqrt{3}, b = \sqrt{3})$ A1A1 N3

(b) attempt to find max (M1)

e.g. setting $f'(x) = 0$, graph

$c = 1.15$ (accept (1.15, 1.13)) A1N2

(c) attempt to substitute either limits or the function into formula M1

e.g. $V = \pi\int_0^c [f(x)]^2 \mathrm{d}x, \pi\int \left[x\ln(4 - x^2)\right]^2, \pi\int_0^{1.149...} y^2 \mathrm{d}x$

$V = 2.16$ A2N2

(d) valid approach recognizing 2 regions (M1)
e.g. finding 2 areas

correct working (A1)

e.g. $\int_0^{-1.73...} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x; -\int_{-1.73...}^{0} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x$

area = 2.07 (accept 2.06) A2N3

[12]

15.) attempt to substitute into formula $V = \int \pi y^2 \mathrm{d}x$ (M1)

integral expression A1

e.g. $\pi\int_0^a (\sqrt{x})^2\, dx, \pi\int x$

correct integration (A1)

e.g. $\int x\mathrm{d}x = \frac{1}{2}x^2$

correct substitution $V = \pi\left[\frac{1}{2}a^2\right]$ (A1)

equating **their** expression to 32π M1

e.g. $\pi\left[\frac{1}{2}a^2\right] = 32\pi$

$a^2 = 64$

$a = 8$ A2 N2

[7]

16.) (a) **METHOD 1**

evidence of substituting $-x$ for x (M1)

$f(-x) = \frac{a(-x)}{(-x)^2 + 1}$ A1

$f(-x) = \frac{-ax}{x^2 + 1}$ $(= -f(x))$ AGN0

METHOD 2

$y = -f(x)$ is reflection of $y = f(x)$ in x axis
and $y = f(-x)$ is reflection of $y = f(x)$ in y axis (M1)

sketch showing these are the same A1

$f(-x) = \frac{-ax}{x^2 + 1}$ $(= -f(x))$ AGN0

(b) evidence of appropriate approach (M1)
e.g. $f'(x) = 0$

to set the numerator equal to 0 (A1)
e.g. $2ax(x^2 - 3) = 0$; $(x^2 - 3) = 0$

$(0, 0)$, $\left(\sqrt{3}, \frac{a\sqrt{3}}{4}\right), \left(-\sqrt{3}, -\frac{a\sqrt{3}}{4}\right)$ (accept $x = 0$, $y = 0$ *etc.*) A1A1A1A1A1N5

(c) (i) correct expression A2

e.g. $\left[\frac{a}{2}\ln(x^2+1)\right]_3^7, \frac{a}{2}\ln 50 - \frac{a}{2}\ln 10, \frac{a}{2}(\ln 50 - \ln 10)$

area = $\frac{a}{2}\ln 5$ A1A1 N2

(ii) **METHOD 1**

recognizing that the shift does not change the area (M1)

e.g. $\int_4^8 f(x-1)dx = \int_3^7 f(x)\mathrm{d}x, \frac{a}{2}\ln 5$

recognizing that the factor of 2 doubles the area (M1)

e.g. $\int_4^8 2f(x-1)\mathrm{d}x = 2\int_4^8 f(x-1)\mathrm{d}x \quad \left(= 2\int_3^7 f(x)\mathrm{d}x\right)$

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ (*i.e.* 2 × **their** answer to (c)(i)) A1N3

METHOD 2

changing variable

let $w = x - 1$, so $\frac{\mathrm{d}w}{\mathrm{d}x} = 1$

$2\int f(w)\mathrm{d}w = \frac{2a}{2}\ln(w^2+1) + c$ (M1)

substituting correct limits

e.g. $\left[a\ln[(x-1)^2+1]\right]_4^8, \left[a\ln(w^2+1)\right]_3^7, a\ln 50 - a\ln 10$ (M1)

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ A1N3

[16]

17.) **Note:** In this question, do not penalize absence of units.

(a) (i) $s = \int(40 - at)\mathrm{d}t$ (M1)

$s = 40t - \frac{1}{2}at^2 + c$ (A1)(A1)

substituting $s = 100$ when $t = 0$ ($c = 100$) (M1)

$s = 40t - \frac{1}{2}at^2 + 100$ A1 N5

(ii) $s = 40t - \frac{1}{2}at^2$ A1 N1

(b) (i) stops at station, so $v = 0$ (M1)

$t = \frac{40}{a}$ (seconds) A1 N2

(ii) evidence of choosing formula for s from (a) (ii) (M1)

substituting $t = \frac{40}{a}$ (M1)

e.g. $40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$

setting up equation M1

e.g. $500 = s$, $500 = 40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}, 500 = \frac{1600}{a} - \frac{800}{a}$

evidence of simplification to an expression which obviously leads to $a = \frac{8}{5}$ A1

e.g. $500a = 800$, $5 = \frac{8}{a}$, $1000a = 3200 - 1600$

$a = \frac{8}{5}$ AGN0

(c) **METHOD 1**

$v = 40 - 4t$, stops when $v = 0$

$40 - 4t = 0$ (A1)

$t = 10$ A1

substituting into expression for s M1

$s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$

$s = 200$ A1

since $200 < 500$ (allow ***FT*** on their s, if $s < 500$) R1

train stops before the station AGN0

METHOD 2

from (b) $t = \frac{40}{4} = 10$ A2

substituting into expression for s

e.g. $s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$ M1

$s = 200$ A1

since $200 < 500$, R1

train stops before the station AGN0

METHOD 3

a is deceleration A2

$4 > \frac{8}{5}$ A1

so stops in shorter time (A1)

so less distance travelled R1

so stops before station AGN0

[17]

18.) (a) finding the limits $x = 0$, $x = 5$ (A1)

integral expression A1

e.g. $\int_0^5 f(x)\mathrm{d}x$

area = 52.1 A1 N2

(b) evidence of using formula $v = \int \pi y^2 \mathrm{d}x$ (M1)

correct expression A1

e.g. volume = $\pi\int_0^5 x^2(x-5)^4\,dx$

volume = 2340 A2N2

(c) area is $\int_0^a x(a-x)\,dx$ A1

$= \left[\frac{ax^2}{2} - \frac{x^3}{3}\right]_0^a$ A1A1

substituting limits (M1)

e.g. $\frac{a^3}{2} - \frac{a^3}{3}$

setting expression equal to area of R (M1)

correct equation A1

e.g. $\frac{a^2}{2} - \frac{a^3}{3}$ = 52.1, $a^3 = 6 \times 52.1$,

$a = 6.79$ A1N3

[14]

19.) (a) finding derivative (A1)

e.g. $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}, \frac{1}{2\sqrt{x}}$

correct value of derivative or its negative reciprocal (seen anywhere) A1

e.g. $\frac{1}{2\sqrt{4}}, \frac{1}{4}$

gradient of normal = $-\frac{1}{\text{gradient of tangent}}$ (seen anywhere) A1

e.g. $-\frac{1}{f'(4)} = -4,\ -2\sqrt{x}$

substituting into equation of line (for normal) M1

e.g. $y - 2 = -4(x - 4)$

$y = -4x + 18$ AGN0

(b) recognition that $y = 0$ at A (M1)

e.g. $-4x + 18 = 0$

$x = \frac{18}{4}\left(= \frac{9}{2}\right)$ A1N2

(c) splitting into two appropriate parts (areas and/or integrals) (M1)

correct expression for area of R A2N3

e.g. area of R = $\int_0^4 \sqrt{x}\,dx + \int_4^{4.5}(-4x+18)\,dx,\ \int_0^4 \sqrt{x}\,dx + \frac{1}{2}\times 0.5 \times 2$ (triangle)

Note: *Award A1 if dx is missing.*

(d) correct expression for the volume from $x = 0$ to $x = 4$ (A1)

e.g. $V = \int_0^4 \pi\left[f(x)^2\right]dx,\ \int_0^4 \pi\sqrt{x}^2\,dx,\ \int_0^4 \pi x\,dx$

$V = \left[\frac{1}{2}\pi x^2\right]_0^4$ A1

$V = \pi\left(\frac{1}{2}\times 16 - \frac{1}{2}\times 0\right)$ (A1)

$V = 8\pi$ A1

finding the volume from $x = 4$ to $x = 4.5$

EITHER

recognizing a cone (M1)

e.g. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2)^2 \times \frac{1}{2}$ (A1)

$= \frac{2\pi}{3}$ A1

total volume is $8\pi + \frac{2}{3}\pi \quad \left(= \frac{26}{3}\pi\right)$ A1N4

OR

$V = \pi\int_4^{4.5}(-4x+18)^2\,dx$ (M1)

$= \int_4^{4.5}\pi(16x^2 - 144x + 324)\,dx$

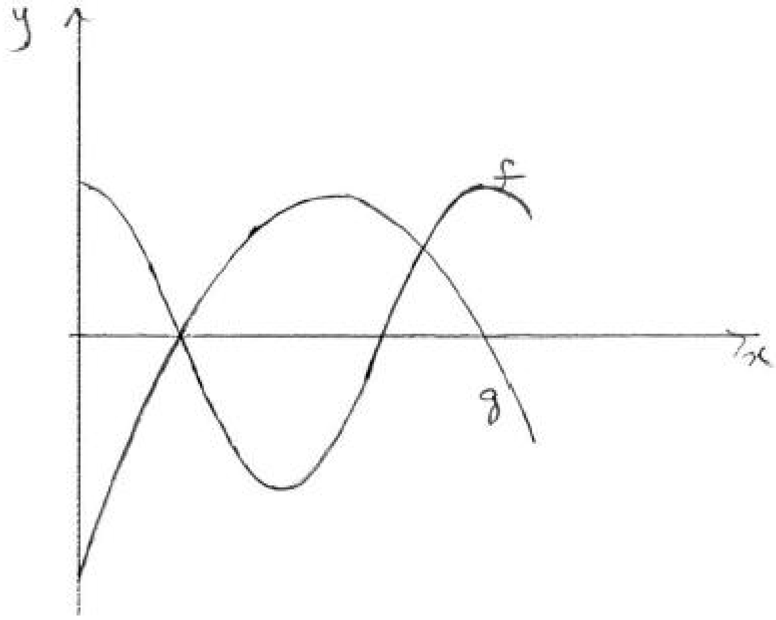
$= \pi\left[\frac{16}{3}x^3 - 72x^2 + 324x\right]_4^{4.5}$ A1

$= \frac{2\pi}{3}$ A1

total volume is $8\pi + \frac{2}{3}\pi \quad \left(= \frac{26}{3}\pi\right)$ A1N4

[17]

20.) (a)



A1A1A1 N3

Note: *Award A1 for f being of sinusoidal shape, with 2 maxima and one minimum,*
A1 for g being a parabola opening down,
A1 for ***two*** *intersection points in approximately correct position.*

(b) (i) (2,0) (accept $x = 2$) A1 N1

(ii) period = 8 A2N2

(iii) amplitude = 5 A1N1

(c) (i) (2, 0), (8, 0) (accept $x = 2$, $x = 8$) A1A1 N1N1

(ii) $x = 5$ (must be an equation) A1N1

(d) **METHOD 1**

intersect when $x = 2$ and $x = 6.79$ (may be seen as limits of integration) A1A1

evidence of approach (M1)

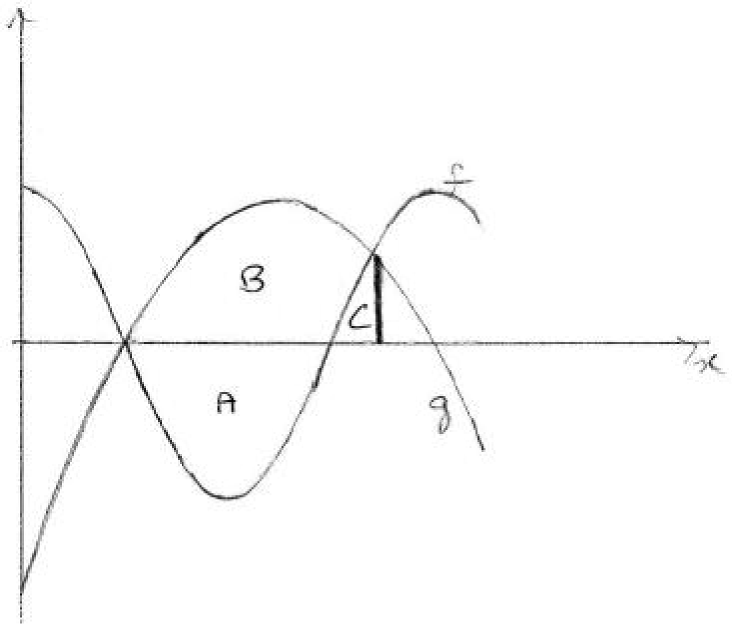
e.g. $\int g - f, \int f(x)\mathrm{d}x - \int g(x)\mathrm{d}x, \int_2^{6.79}\left((-0.5x^2 + 5x - 8 - \left(5\cos\frac{\pi}{4}x\right)\right)$

area = 27.6 A2N3

METHOD 2

intersect when $x = 2$ and $x = 6.79$ (seen anywhere) A1A1

evidence of approach using a sketch of g and f, or $g - f$. (M1)



e.g. area $A + B - C$, 12.7324 + 16.0938 – 1.18129...
area = 27.6 A2N3

[15]

21.) (a) $\int \frac{1}{2x+3}\,dx = \frac{1}{2}\ln(2x+3) + C$ $\left(\text{accept } \frac{1}{2}\ln|(2x+3)| + C\right)$ A1A1 N2

(b) $\int_0^3 \frac{1}{2x+3}\,dx = \left[\frac{1}{2}\ln(2x+3)\right]_0^3$

evidence of substitution of limits (M1)

e.g. $\frac{1}{2}\ln 9 - \frac{1}{2}\ln 3$

evidence of correctly using $\ln a - \ln b = \ln \frac{a}{b}$ (seen anywhere) (A1)

e.g. $\frac{1}{2}\ln 3$

evidence of correctly using $a \ln b = \ln b^a$ (seen anywhere) (A1)

e.g. $\ln\sqrt{\frac{9}{3}}$

$P = 3$ (accept $\ln\sqrt{3}$) A1 N2

[6]

22.) evidence of anti-differentiation (M1)

e.g. $s = \int (6e^{3x} + 4)\,dx$

$s = 2e^{3t} + 4t + C$ A2A1

substituting $t = 0$, (M1)

$7 = 2 + C$ A1

$C = 5$

$s = 2e^{3t} + 4t + 5$ A1 N3

[7]

23.) (a) evidence of factorizing 3/division by 3 A1

e.g. $\int_1^5 3f(x)\mathrm{d}x = 3\int_1^5 f(x)\mathrm{d}x$, $\frac{12}{3}$, $\int_1^5 \frac{3f(x)\mathrm{d}x}{3}$

(do not accept 4 as this is show that)

evidence of stating that reversing the limits changes the sign A1

e.g. $\int_5^1 f(x)\mathrm{d}x = -\int_1^5 f(x)\mathrm{d}x$

$\int_5^1 f(x)\mathrm{d}x = -4$ AG N0

(b) evidence of correctly combining the integrals (seen anywhere) (A1)

e.g. $I = \int_1^2 (x + f(x))\mathrm{d}x + \int_2^5 (x + f(x))\mathrm{d}x = \int_1^5 (x + f(x))\mathrm{d}x$

evidence of correctly splitting the integrals (seen anywhere) (A1)

e.g. $I = \int_1^5 x\mathrm{d}x + \int_1^5 f(x)\mathrm{d}x$

$\int x\mathrm{d}x = \frac{x^2}{2}$ (seen anywhere) A1

$\int_1^5 x\mathrm{d}x = \left[\frac{x^2}{2}\right]_1^5 = \frac{25}{2} - \frac{1}{2} \left(= \frac{24}{2}, 12\right)$ A1

$I = 16$ A1 N3

[7]

24.) (a) (i) range of f is $[-1, 1]$, $(-1 \le f(x) \le 1)$ A2 N2

(ii) $\sin^3 x = 1 \Rightarrow \sin x = 1$ A1

justification for one solution on $[0, 2\pi]$ R1

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere) A1 N1

(b) $f'(x) = 3\sin^2 x \cos x$ A2 N2

(c) using $V = \int_a^b \pi y^2 \mathrm{d}x$ (M1)

$$V = \int_0^{\frac{\pi}{2}} \pi \left(\sqrt{3} \sin x \cos^{\frac{1}{2}} x \right)^2 \mathrm{d}x$$ (A1)

$$= \pi \int_0^{\frac{\pi}{2}} 3 \sin^2 x \cos x \,\mathrm{d}x$$ A1

$$V = \pi \left[\sin^3 x \right]_0^{\frac{\pi}{2}} \quad \left(= \pi \left(\sin^3 \left(\frac{\pi}{2} \right) - \sin^3 0 \right) \right)$$ A2

evidence of using $\sin \frac{\pi}{2} = 1$ and $\sin 0 = 0$ (A1)

e.g. $\pi(1 - 0)$

$V = \pi$ A1 N1

[14]

25.) (a) (i) intersection points $x = 3.77$, $x = 8.30$ (may be seen as the limits) (A1)(A1)

approach involving subtraction and integrals (M1)

fully correct expression A2

e.g. $\int_{3.77}^{8.30} ((-4\cos(0.5x) + 2) - (\ln(3x - 2) + 1)) \mathrm{d}x$,

$\int_{3.77}^{8.30} g(x) \mathrm{d}x - \int_{3.77}^{8.30} f(x) \mathrm{d}x$ N5

(ii) $A = 6.46$ A1 N1

(b) (i) $f'(x) = \frac{3}{3x - 2}$ A1A1 N2

Note: *Award A1 for numerator (3), A1 for denominator (3x - 2), but penalize 1 mark for additional terms.*

(ii) $g'(x) = 2 \sin (0.5x)$ A1A1 N2

Note: *Award A1 for 2, A1 for sin (0.5x), but penalize 1 mark for additional terms.*

(c) evidence of using derivatives for gradients (M1)

correct approach (A1)

e.g. $f'(x) = g'(x)$, points of intersection

$x = 1.43$, $x = 6.10$ A1A1 N2N2

[14]

26.) (a) evidence of using the product rule M1

$f'(x) = \mathrm{e}^x(1 - x^2) + \mathrm{e}^x(-2x)$ A1A1

Note: *Award A1 for* $e^x(1 - x^2)$, *A1 for* $e^x(-2x)$.

$f'(x) = \mathrm{e}^x(1 - 2x - x^2)$ AG N0

(b) $y = 0$ A1 N1

(c) at the local maximum or minimum point

$f'(x) = 0$ $(e^x(1 - 2x - x^2) = 0)$ (M1)

$\Rightarrow 1 - 2x - x^2 = 0$ (M1)

$r = -2.41$ $s = 0.414$ A1A1 N2N2

(d) $f'(0) = 1$ A1

gradient of the normal = –1 A1

evidence of substituting into an equation for a straight line (M1)

correct substitution A1

e.g. $y - 1 = -1(x - 0)$, $y - 1 = -x$, $y = -x + 1$

$x + y = 1$ AG N0

(e) (i)intersection points at $x = 0$ and $x = 1$ (may be seen as the limits) (A1)

approach involving subtraction and integrals (M1)

fully correct expression A2 N4

e.g. $\int_0^1 \left(e^x\left(1 - x^2\right) - (1 - x)\right)dx, \int_0^1 f(x)dx - \int_0^1 (1 - x)dx$

(ii) area $R = 0.5$ A1 N1

[17]

27.) (a) substituting $t = 0$ (M1)
e.g. $a(0) = 0 + \cos 0$
$a(0) = 1$ A1 N2

(b) evidence of integrating the acceleration function (M1)
e.g. $\int (2t + \cos t)dt$

correct expression $t^2 + \sin t + c$ A1A1

Note: *If "+c" is omitted, award no further marks.*

evidence of substituting (0, 2) into indefinite integral (M1)
e.g. $2 = 0 + \sin 0 + c$, $c = 2$
$v(t) = t^2 + \sin t + 2$ A1 N3

(c) $\int (t^2 + \sin t + 2)dt = \frac{t^3}{3} - \cos t + 2t$ A1A1A1

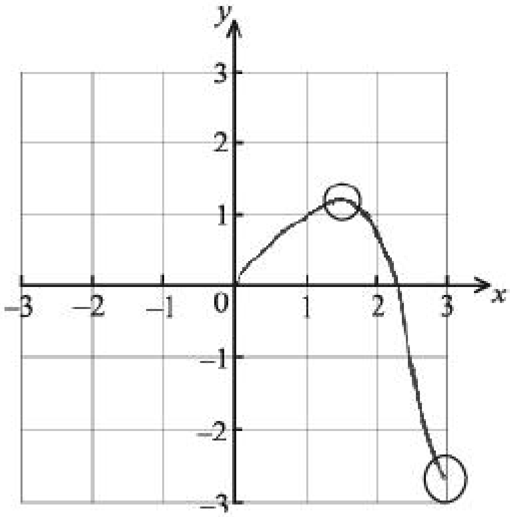
Note: *Award A1 for each correct term.*

evidence of using $v(3) - v(0)$ (M1)
correct substitution A1
e.g. $(9 - \cos 3 + 6) - (0 - \cos 0 + 0)$, $(15 - \cos 3) - (-1)$
$16 - \cos 3$ (accept $p = 16$, $q = -1$) A1A1 N3

(d) reference to motion, reference to first 3 seconds R1R1 N2
e.g. displacement in 3 seconds, distance travelled in 3 seconds

[16]

28.) (a)



A1A2 N3

Notes: *Award **A1** for correct domain, $0 \le x \le 3$.*
*Award **A2** for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2.*

(b) $a = 2.31$ A1 N1

(c) evidence of using $V = \pi \int [f(x)]^2 \, dx$ (M1)

fully correct integral expression A2

e.g. $V = \pi \int_0^{2.31} [x \cos(x - \sin x)]^2 dx, V = \pi \int_0^{2.31} [f(x)]^2 dx$

$V = 5.90$ A1 N2

[8]

29.)

(a) correctly finding the derivative of e^{2x}, *i.e.* $2e^{2x}$ A1

correctly finding the derivative of $\cos x$, *i.e.* $-\sin x$ A1

evidence of using the product rule, seen anywhere M1

e.g. $f'(x) = 2e^{2x} \cos x - e^{2x} \sin x$

$f'(x) = e^{2x}(2 \cos x - \sin x)$ AG N0

(b) evidence of finding $f(0) = 1$, seen anywhere A1

attempt to find the gradient of f (M1)
e.g. substituting $x = 0$ into $f'(x)$

value of the gradient of f A1
e.g. $f'(0) = 2$, equation of tangent is $y = 2x + 1$

gradient of normal $= -\frac{1}{2}$ (A1)

$y - 1 = -\frac{1}{2}x$ $\left(y = -\frac{1}{2}x + 1\right)$ A1 N3

(c) (i) evidence of equating correct functions M1

e.g. $e^{2x}\cos x = -\frac{1}{2}x + 1$, sketch showing intersection of graphs

$x = 1.56$ A1 N1

(ii) evidence of approach involving subtraction of integrals/areas (M1)

e.g. $\int [f(x) - g(x)]\mathrm{d}x, \int f(x)\mathrm{d}x$ – area under trapezium

fully correct integral expression A2

e.g. $\int_0^{1.56}\left[e^{2x}\cos x - \left(-\frac{1}{2}x + 1\right)\right]\mathrm{d}x, \int_0^{1.56} e^{2x}\cos x\mathrm{d}x - 0.951...$

area = 3.28 A1 N2

[14]

30.) (a) $\int_1^2 (3x^2 - 2)\mathrm{d}x = \left[x^3 - 2x\right]_1^2$ A1A1

= (8 – 4) – (1 – 2) (A1)

= 5 A1 N2

(b) $\int_0^1 2e^{2x}\mathrm{d}x = \left[e^{2x}\right]_0^1$ A1

$= e^2 - e^0$ (A1)

$= e^2 - 1$ A1N2

[7]

31.) (a) $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ (M1)

$= -10$ (m s^{-2}) A1 N2

(b) $s = \int v\ \mathrm{d}t$ (M1)

$= 50t - 5t^2 + c$ A1

$40 = 50(0) - 5(0) + c \Rightarrow c = 40$ A1

$s = 50t - 5t^2 + 40$ A1N2

Note: *Award **(M1)** and the first **A1** in part (b) if c is missing, but do **not** award the final 2 marks.*

[6]

32.) (a) period = $\frac{2\pi}{2} = \pi$ M1A1 N2

(b) $m = \frac{\pi}{2}$ A2N2

(c) Using $A = \int_0^{\frac{\pi}{2}} \sin 2x\mathrm{d}x$ (M1)

Integrating correctly, $A = \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$ A1

Substituting, $A = -\frac{1}{2}\cos\pi - (-\frac{1}{2}\cos 0)$ (M1)

Correct values, $A = -\frac{1}{2}(-1) - (-\frac{1}{2}(1)) \quad \left(= \frac{1}{2} + \frac{1}{2}\right)$ A1A1

$A = 1$ A1N2

[10]

33.) (a) Using the chain rule (M1)
$f'(x) = (2\cos(5x - 3))5$ $(= 10\cos(5x - 3))$ A1
$f''(x) = -(10\sin(5x - 3))5$
$= -50\sin(5x - 3)$ A1A1 N2

Note: *Award A1 for sin (5x – 3), A1 for –50.*

(b) $\int f(x)\mathrm{d}x = -\frac{2}{5}\cos(5x - 3) + c$ A1A1N2

Note: *Award A1 for cos(5x – 3), A1 for* $-\frac{2}{5}$.

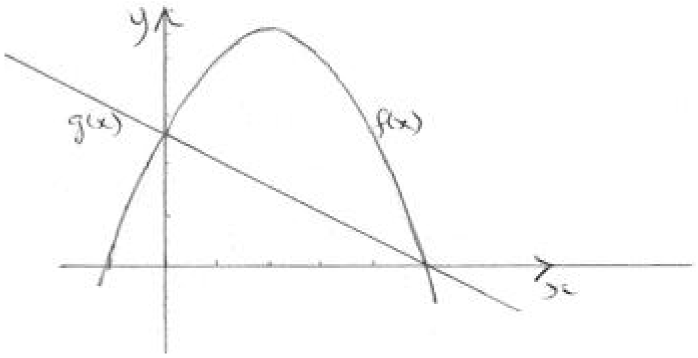
[6]

34.) (a) Curve intersects y-axis when $x = 0$ (A1)
Gradient of tangent at y-intercept = 2 A1
$\Rightarrow$ gradient of $N = -\frac{1}{2}(= -0.5)$ A1
Finding y-intercept, 2.5 A1
Therefore, equation of N is $y = -0.5x + 2.5$ AG N0

(b) N intersects curve when $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ A1
Solving equation (M1)
e.g. sketch, factorising
$\Rightarrow x = 0$ or $x = 5$ A1

Other point when $x = 5$ (R1)
$x = 5 \Rightarrow y = 0$ (so other point (5, 0)) A1N2

(c)



Using appropriate method, with subtraction/correct expression, **correct** limitsM1A1
e.g. $\int_0^5 f(x)\mathrm{d}x - \int_0^5 g(x)\mathrm{d}x$, $\int_0^5 (-0.5x^2 + 2.5x)\mathrm{d}x$
Area = 10.4 A2N2

[13]

35.) Evidence of integration (M1)

$s = -0.5\,\mathrm{e}^{-2t} + 6t^2 + c$ A1A1

Substituting $t = 0$, $s = 2$ (M1)

eg $2 = -0.5 + c$

$c = 2.5$ (A1)

$s = -0.5\,e^{-2t} + 6t^2 + 2.5$ A1 N4

[6]

36.) (a) 10 A1 N1

(b) $\int_1^3 3x^2 + f(x)\,dx = \int_1^3 3x^2 dx + \int_1^3 f(x)\,dx$

$\int_1^3 3x^2 dx = [x^3]_1^3 = 27 - 1$ (A1)

$= 26$ (may be seen later) A1

Splitting the integral (seen anywhere) M1

e.g. $\int 3x^2 dx + \int f(x)\,dx$

Using $\int_1^3 f(x)\,dx = 5$ (M1)

eg $\int_1^3 3x^2 + f(x)\,dx = 26 + 5$

$\int_1^3 3x^2 + f(x)\,dx = 31$ A1 N3

[6]

37.) $f(x) = \int (12x^2 - 2)\,dx$ (M1)

$f(x) = 4x^3 - 2x + c$ A1A1

Substituting $x = -1$, $y = 1$ (M1)

eg $1 = 4(-1)^3 - 2(-1) + c$

$c = 3$ (A1)

$f(x) = 4x^3 - 2x + 3$ A1 N4

[6]

38.) (a) π (3.14) (accept (π, 0), (3.14, 0)) A1 N1

(b) (i) For using the product rule (M1)

$f'(x) = e^x \cos x + e^x \sin x = e^x(\cos x + \sin x)$ A1A1 N3

(ii) At B, $f'(x) = 0$ A1 N1

(c) $f''(x) = e^x \cos x - e^x \sin x + e^x \sin x + e^x \cos x$ A1A1

$= 2e^x \cos x$ AG N0

(d) (i) At A, $f''(x) = 0$ A1 N1

(ii) Evidence of setting up **their** equation (may be seen in part (d)(i)) A1

eg $2e^x \cos x = 0$, $\cos x = 0$

$x = \frac{\pi}{2}\ (=1.57)$, $y = e^{\frac{\pi}{2}}\ (=4.81)$ A1A1

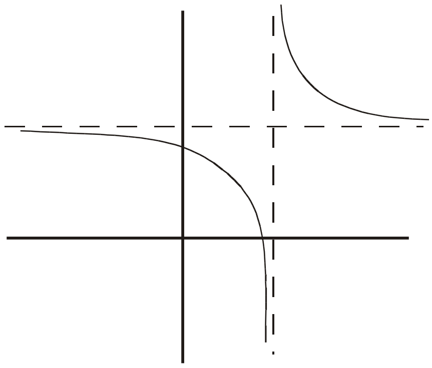
Coordinates are $\left(\frac{\pi}{2}, e^{\frac{\pi}{2}}\right)(1.57, 4.81)$ N2

(e) (i) $\int_0^{\pi} e^x \sin x \, dx$ or $\int_0^{\pi} f(x) dx$ A2 N2

(ii) Area = 12.1 A2 N2

[15]

39.) (a)



A1A1A1 N3

***Notes**: Award A1 for **both** asymptotes shown. The asymptotes need not be labelled.*
*Award A1 for the left branch in **approximately** correct position,*
*A1 for the right branch in **approximately** correct position.*

(b) (i) $y = 3$, $x = \frac{5}{2}$ (must be equations) A1A1 N2

(ii) $x = \frac{14}{6}$ $\left(\frac{7}{3} \text{ or } 2.33, \text{ also accept} \left(\frac{14}{6}, 0\right)\right)$ A1 N1

(iii) $y = \frac{14}{6}$ $(y = 2.8)$ $\left(\text{accept} \left(0, \frac{14}{5}\right) \text{or} (0, 2.8)\right)$ A1 N1

(c) (i) $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx = 9x +$

$3\ln(2x-5) - \frac{1}{2(2x-5)} + C$ A1A1A1

A1A1 N5

(ii) Evidence of using $V = \int_a^b \pi y^2 dx$ (M1)

Correct expression A1

eg $\int_3^a \pi\left(3+\frac{1}{2x-5}\right)^2 \mathrm{d}x,\ \pi\int_3^a\left(9+\frac{6}{2x-5}+\frac{1}{(2x-5)^2}\right)\mathrm{d}x,$

$\left[9x+3\ln(2x-5)-\frac{1}{2(2x-5)}\right]_3^a$

Substituting $\left(9a+3\ln(2a-5)-\frac{1}{2(2a-5)}\right)-\left(27+3\ln 1-\frac{1}{2}\right)$ A1

Setting up an equation (M1)

$9a-\frac{1}{2(2a-5)}-27+\frac{1}{2}+3\ln(2a-5)-3\ln 1=\left(\frac{28}{3}+3\ln 3\right)$

Solving gives $a = 4$ A1 N2

[17]

40.) (a) (i) $p = 2$ A1 N1

(ii) $q = 1$ A1 N1

(b) (i) $f(x) = 0$ (M1)

$2 - \frac{3x}{x^2-1} = 0 \quad (2x^2 - 3x - 2 = 0)$ A1

$x = -\frac{1}{2}$ $x = 2$

$\left(-\frac{1}{2}, 0\right)$ A1 N2

(ii) Using $V = \int_a^b \pi y^2 \mathrm{d}x$ (limits not required) (M1)

$V = \int_{\frac{1}{2}}^{0} \pi\left(2-\frac{3x}{x^2-1}\right)^2 \mathrm{d}x$ A2

$V = 2.52$ A1 N2

(c) (i) Evidence of appropriate method M1

eg Product or quotient rule

Correct derivatives of $3x$ and $x^2 - 1$ A1A1

Correct substitution A1

eg $\frac{-3(x^2-1)-(-3x)(2x)}{(x^2-1)^2}$

$f'(x) = \frac{-3x^2+3+6x^2}{(x^2-1)^2}$ A1

$f'(x) = \frac{3x^2+3}{(x^2-1)^2} = \frac{3(x^2+1)}{(x^2-1)^2}$ AG N0

(ii) **METHOD 1**

Evidence of using $f'(x) = 0$ at max/min (M1)

$3(x^2 + 1) = 0$ $(3x^2 + 3 = 0)$ A1

no (real) solution R1

Therefore, no maximum or minimum. AG N0

METHOD 2

Evidence of using $f'(x) = 0$ at max/min (M1)

Sketch of $f'(x)$ with good asymptotic behaviour A1

Never crosses the x-axis R1

Therefore, no maximum or minimum. AG N0

METHOD 3

Evidence of using $f'(x) = 0$ at max/min (M1)

Evidence of considering the sign of $f'(x)$ A1

$f'(x)$ is an increasing function ($f'(x) > 0$, always) R1

Therefore, no maximum or minimum. AG N0

(d) For using integral (M1)

Area = $\int_0^a g(x)\,dx$ $\left(\text{or} \int_0^a f'(x)\,dx \text{ or} \int_0^a \frac{3x^2+3}{(x^2-1)^2}\,dx\right)$ A1

Recognizing that $\int_0^a g(x)\,dx = f(x)\Big|_0^a$ A2

Setting up equation (seen anywhere) (M1)

Correct equation A1

eg $\int_0^a \frac{3x^2+3}{(x^2-1)^2}\,dx = 2$, $\left[2 - \frac{3a}{a^2-1}\right] - [2-0] = 2$, $2a^2 + 3a - 2 = 0$

$a = \frac{1}{2}$ $a = -2$

$a = \frac{1}{2}$ A1 N2

[24]

41.) (a) $\int_{\frac{3\pi}{2}}^{2\pi} \cos x\,dx$ A1 N1

(b) Area of A = 1 A1 N1

(c) Evidence of attempting to find the area of B (M1)

eg $\int_{\frac{4\pi}{3}}^{\frac{3\pi}{2}} y\,dx$, -0.134

Evidence of recognising that area B is under the curve/integral is negative (M1)

$$eg \; -\int_{\frac{4\pi}{3}}^{\frac{3\pi}{2}} y\,\mathrm{d}x, \int_{\frac{3\pi}{2}}^{\frac{4\pi}{3}} \cos x\,\mathrm{d}x, \left|\int_{\frac{4\pi}{3}}^{\frac{3\pi}{2}} \cos x\,\mathrm{d}x\right|$$

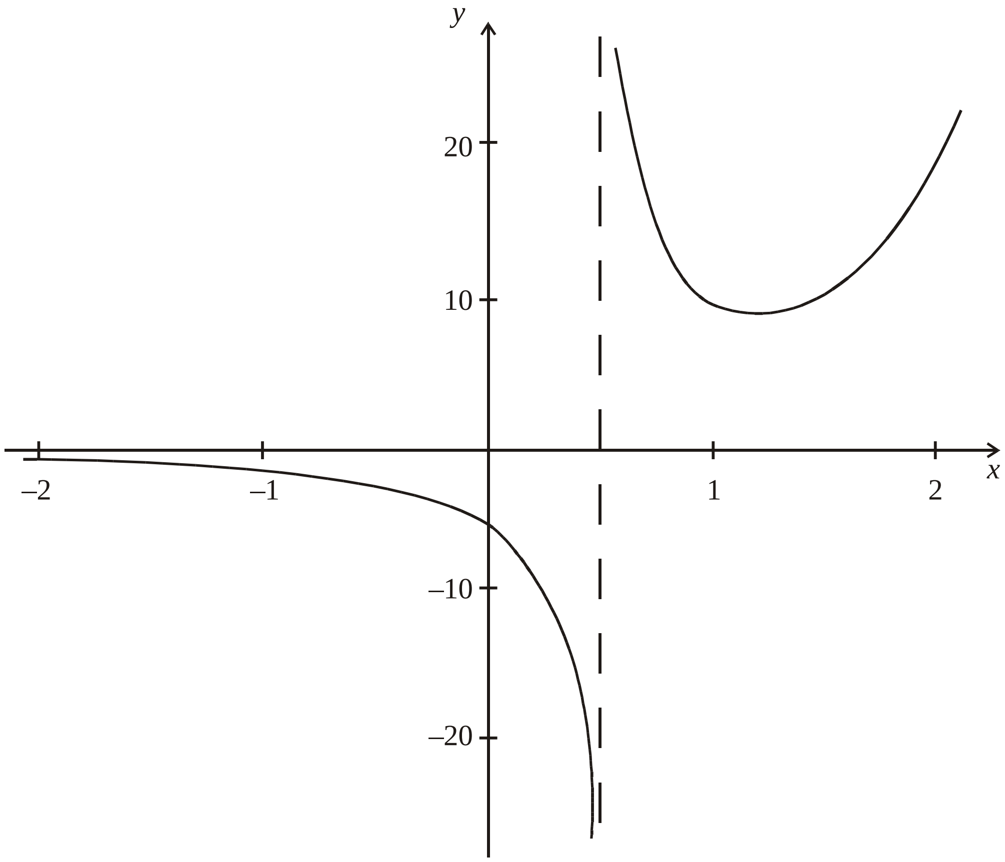
Area of B = 0.134 $\left(\text{accept } \frac{2-\sqrt{3}}{2}\right)$ (A1)

Total Area = 1 + 0.134

= 1.13 $\left(\text{accept } \frac{4-\sqrt{3}}{2}\right)$ A1 N4

[6]

42.) (a)



A1A1A1 N3

Note: *Award A1 for the left branch asymptotic to the x-axis and crossing the y-axis,*
A1 for the right branch approximately the correct shape,
A1 for a vertical asymptote at approximately x = $\frac{1}{2}$.

(b) (i) $x = \frac{1}{2}$ (must be an equation) A1 N1

(ii) $\int_0^2 f(x)\,\mathrm{d}x$ A1 N1

(iii) Valid reason R1 N1

eg reference to area undefined or discontinuity

Note: *GDC reason* **not** *acceptable.*

(c) (i) $V = \pi \int_1^{1.5} f(x)^2 \, dx$ A2 N2

(ii) $V = 105$ (accept 33.3π) A2 N2

(d) $f'(x) = 2e^{2x-1} - 10(2x-1)^{-2}$ A1A1A1A1 N4

(e) (i) $x = 1.11$ (accept (1.11, 7.49)) A1 N1

(ii) $p = 0, q = 7.49$ (accept $0 \le k < 7.49$) A1A1 N2

[17]

43.) (a) Attempting to use the formula $V = \int_a^b \pi y^2 dx$ (M1)

Volume = $\pi \int_0^2 \left(2x - x^2\right)^2 dx$ A2 N3

(b) Volume = $\pi \int_0^2 \left(4x^2 - 4x^3 + x^4\right) dx$ (A1)

$= \pi \left[4\frac{x^3}{3} - 4\frac{x^4}{4} + \frac{x^5}{5} \right]_0^2$ (A1)

$= \frac{16\pi}{15}$ or 3.35 (accept 1.07π) A1 N3

[6]

44.) (a) (i) $f'(x) = -\frac{3}{2}x + 1$ A1A1 N2

(ii) For using the derivative to find the gradient of the tangent (M1)

$f'(2) = -2$ (A1)

Using negative reciprocal to find the gradient of the normal $\left(\frac{1}{2}\right)$ M1

$y - 3 = \frac{1}{2}(x-2)$ $\left(\text{or } y = \frac{1}{2}x + 2\right)$ A1 N3

(iii) Equating $-\frac{3}{4}x^2 + x + 4 = \frac{1}{2}x + 2$ (or sketch of graph) M1

$3x^2 - 2x - 8 = 0$ (A1)

$(3x + 4)(x - 2) = 0$

$x = -\frac{4}{3}$ $(= -1.33)$ (accept $\left(-\frac{4}{3}, \frac{4}{3}\right)$ or $x = -\frac{4}{3}, x = 2$) A1 N2

(b) (i)Any **completely** correct expression (accept absence of dx) A2 *eg* $\int_{-1}^{2}\left(-\frac{3}{4}x^2+x+4\right)\mathrm{d}x, \left[-\frac{1}{4}x^3+\frac{1}{2}x^2+4x\right]_{-1}^{2}$ N2

(ii) Area = $\frac{45}{4}(=11.25)$ (accept 11.3) A1 N1

(iii) Attempting to **use** the formula for the volume (M1)

eg $\int_{-1}^{2}\pi\left(-\frac{3}{4}x^2+x+4\right)\mathrm{d}x, \pi\int_{-1}^{2}\left(-\frac{3}{4}x^2+x+4\right)^2\mathrm{d}x$ A2 N3

(c) $\int_{1}^{k} f(x)\,\mathrm{d}x=\left[-\frac{1}{4}x^3+\frac{1}{2}x^2+4x\right]_{1}^{k}$ A1A1A1

Note: *Award A1 for* $-\frac{1}{4}x^3$*, A1 for* $\frac{1}{2}x^2$*, A1 for 4x.*

Substituting $\left(-\frac{1}{4}k^3+\frac{1}{2}k^2+4k\right)-\left(-\frac{1}{4}+\frac{1}{2}+4\right)$ (M1)(A1)

$= -\frac{1}{4}k^3+\frac{1}{2}k^2+4k-4.25$ A1 N3

[21]

45.) (a) **METHOD 1**

Attempting to interchange x and y (M1)

Correct expression $x = 3y - 5$ (A1)

$f^{-1}(x)=\frac{x+5}{3}$ A1 N3

METHOD 2

Attempting to solve for x in terms of y (M1)

Correct expression $x = \frac{y+5}{3}$ (A1)

$f^{-1}(x)=\frac{x+5}{3}$ A1 N3

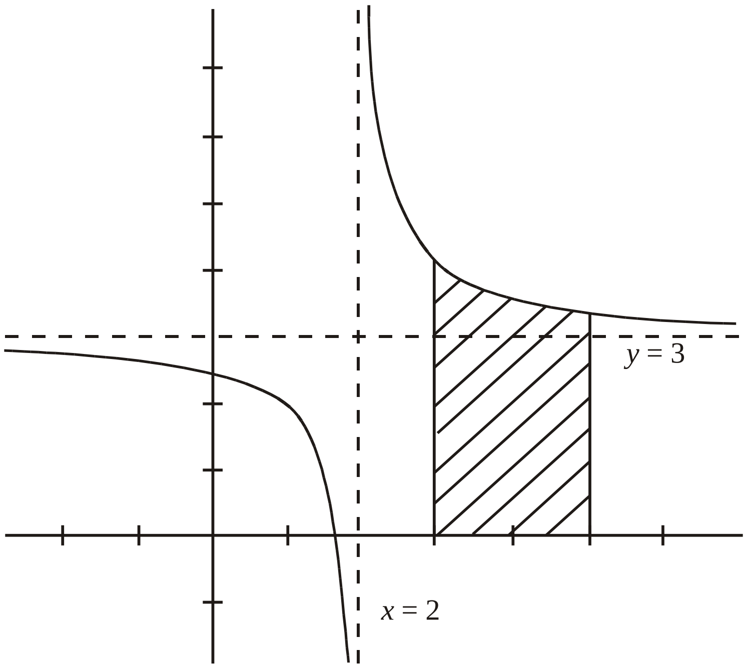
(b) For correct composition $(g^{-1}\circ f)(x) = (3x - 5) + 2$ (A1)

$(g^{-1}\circ f)(x) = 3x - 3$ A1 N2

(c) $\frac{x+3}{3} = 3x - 3 \; (x + 3 = 9x - 9)$ (A1)

$x = \frac{12}{8}$ A1 N2

(d) (i)



A1A1A1 N3

***Note**: Award A1 for approximately correct x and y intervals, A1 for two branches of correct shape, A1 for both asymptotes.*

(ii) (Vertical asymptote) $x = 2$, (Horizontal asymptote) $y = 3$ A1A1 N2

(Must be equations)

(e) (i) $3x + \ln(x-2) + C (3x + \ln|x-2| + C)$ A1A1 N2

(ii) $[3x + \ln(x-2)]_3^5$ (M1)

$= (15 + \ln 3) - (9 + \ln 1)$ A1

$= 6 + \ln 3$ A1 N2

(f) Correct shading (see graph). A1 N1

[18]

46.) $s = \int v \, dt$ (M1)

$s = \frac{1}{2}e^{2t-1} + c$ A1A1

Substituting $t = 0.5$

$\frac{1}{2} + c = 10$

$c = 9.5$ (A1)

Substituting $t = 1$ M1

$s = \frac{1}{2}e + 9.5 (= 10.9 \text{ to } 3 \, s.f.)$ A1 N3

[6]

47.) Using $V = \int \pi y^2 \, dx$ (M1)

Correctly integrating $\int \left(x^{\frac{1}{2}}\right)^2 dx = \frac{x^2}{2}$ A1

$V = \pi \left[\frac{x^2}{2}\right]_0^a$ A1

$= \frac{\pi a^2}{2}$ (A1)

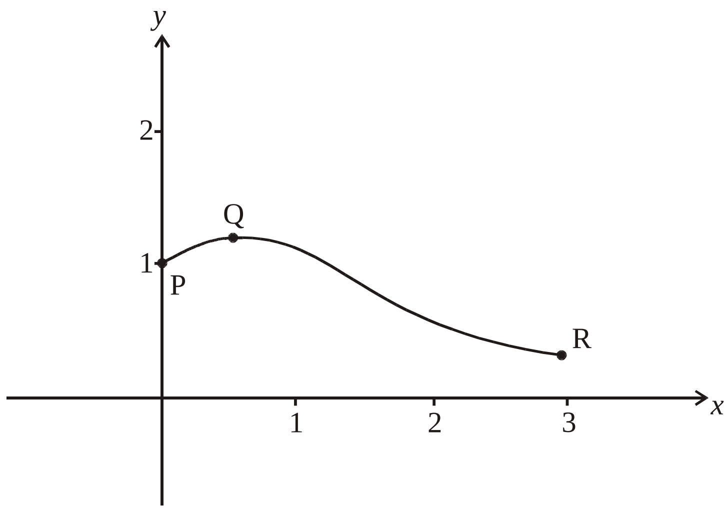
Setting up **their** equation $\left(\frac{1}{2}\pi a^2 = 0.845\pi\right)$ M1

$a^2 = 1.69$

$a = 1.3$ A1 N2

[6]

48.) (a)



A1A1A1 N3

***Note**: Award A1 for the shape of the curve,*
A1 for correct domain,
*A1 for labelling **both** points P and Q in approximately correct positions.*

(b) (i) Correctly finding derivative of $2x + 1$ *ie* 2 (A1)

Correctly finding derivative of e^{-x} *ie* $-e^{-x}$ (A1)

Evidence of using the product rule (M1)

$f'(x) = 2e^{-x} + (2x + 1)(-e^{-x})$ A1

$= (1 - 2x)e^{-x}$ AG N0

(ii) At **Q**, $f'(x) = 0$ (M1)

$x = 0.5$, $y = 2e^{-0.5}$ A1A1

Q is $(0.5, 2e^{-0.5})$ N3

(c) $1 \le k < 2e^{-0.5}$ A2 N2

(d) Using $f''(x) = 0$ at the point of inflexion M1

$e^{-x}(-3 + 2x) = 0$

This equation has only one root. R1

So f has only one point of inflexion. AG N0

(e) At R, $y = 7e^{-3}$ (= 0.34850 ...) (A1)

Gradient of (PR) is $\frac{7e^{-3} - 1}{3}\,(= -0.2172)$ (A1)

Equation of (PR) is $g(x) = \left(\frac{7e^{-3} - 1}{3}\right)x + 1\,(= -0.2172x + 1)$ A1

Evidence of appropriate method, involving subtraction of integrals or areas M2

Correct limits/endpoints A1

eg $\int_0^3 (f(x) - g(x))\,dx$, area under curve − area under PR

Shaded area is $\int_0^3 \left((2x+1)e^{-x} - \left(\frac{7e^{-3} - 1}{3}x + 1\right)\right)dx$

= 0.529 A1 N4

[21]

49.) (a) Using the chain rule (M1)

$f'(x) = (2\cos(5x - 3))\,5\ (= 10\cos(5x - 3))$ A1

$f''(x) = -(10\sin(5x - 3))\,5$

$= -50\sin(5x - 3)$ A1A1 4

***Note**: Award (A1) for sin (5x – 3), (A1) for –50.*

(b) $\int f(x)dx = \frac{2}{5}\cos(5x - 3) + c$ A1A1 2

***Note**: Award (A1) for cos (5x–3), (A1) for* $-\frac{2}{5}$.

[6]

50.) (a) $a = \frac{dv}{dt}$ (M1)

$= -10$ A1 3

(b) $s = \int v\,dt$ (M1)

$= 50t - 5t^2 + c$ A1

$40 = 50(0) - 5(0) + c \Rightarrow c = 40$ A1

$s = 50t - 5t^2 + 40$ A1 3

***Note**: Award (M1) and the first (A1) in part (b) if c is missing,* *but do not award the final 2 marks.*

[6]

51.) (a) (i) $f'(x) = -x + 2$ A1

(ii) $f'(0) = 2$ A1 2

(b) Gradient of tangent at y-intercept = $f'(0) = 2$

$\Rightarrow$ gradient of normal = $\frac{1}{2}$ (= –0.5) A1

Finding y-intercept is 2.5 A1

Therefore, equation of the normal is

$y - 2.5 = \sim(x - 0)$ ($y - 2.5 = -0.5x$) M1

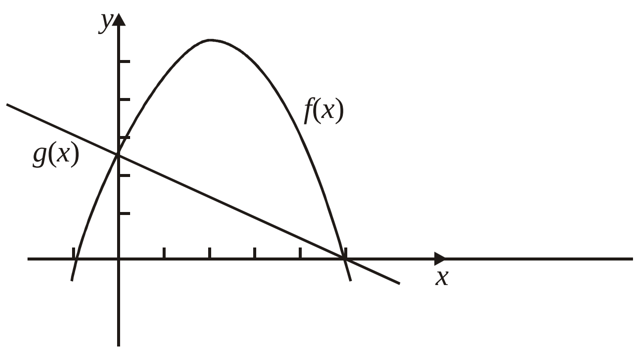
($y = -0.5x + 2.5$ (AG) 3

(c) (i) **EITHER**

solving $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ (M1)A1

$\Rightarrow x = 0$ or $x = 5$ A1 2

OR



M1

Curves intersect at $x = 0$, $x = 5$ (A1)

So solutions to $f(x) = g(x)$ are $x = 0$, $x = 5$ A1 2

OR

$\Rightarrow 0.5x^2 - 2.5x = 0$ (A1)

$\Rightarrow -0.5x(x - 5) = 0$ M1

$\Rightarrow x = 0$ or $x = 5$ A1 2

(ii) Curve and normal intersect when $x = 0$ or $x = 5$ (M2)

Other point is when $x = 5$

$\Rightarrow y = -0.5(5) + 2.5 = 0$ (so other point (5, 0) A1 2

(d) (i)Area = $\int_0^5 (f(x) - g(x))\,dx \left(\text{or } \int_0^5 (-0.5x^2 + 2x + 2.5)\,dx - \frac{1}{2} \times 5 \times 2.5\right)$

A1A1A1 3

***Note**: Award (A1) for the integral, (A1) for both correct limits on the integral, and (A1) for the difference.*

(ii) Area = Area under curve – area under line ($A = A_1 - A_2$) (M1)

(A1) = $\frac{50}{3}$, $A_2 = \frac{25}{4}$

Area = $\frac{50}{3} - \frac{25}{4} = \frac{125}{12}$ (or 10.4 (3sf) A1 2

[16]

52.) (a) (i) $p = (10x + 2) – (1 + e^{2x})$ A2 2

***Note**: Award (A1) for $(l + e^{2x}) – (10x + 2)$.*

(ii) $\frac{dp}{dx} = 10 – 2e^{2x}$ A1A1

$\frac{dp}{dx} = 0 \quad (10 – 2e^{2x} = 0)$ M1

$x = \frac{1n\,5}{2}$ (= 0.805) A1 4

(b) (i) **METHOD 1**

$x = 1 + e^{2y}$ M1

$1n(x – 1) = 2y$ A1

$f^{-1}(x) = \frac{1n(x-1)}{2} \left(\text{Allow } y = \frac{1n(x-1)}{2}\right)$ A1 3

METHOD 2

$y – 1 = e^{2x}$ A1

$\frac{\ln(y-1)}{2} = x$ M1

$f^{-1}(x) = \frac{1n(x-1)}{2} \left(\text{Allow } y = \frac{1n(x-1)}{2}\right)$ A1 3

(ii) $a = \frac{1n(5-1)}{2}\left(=\frac{1}{2}1n2^2\right)$ M1

$= \frac{1}{2} \times 21n2$ A1

$= 1n\,2$ AG 2

(c) Using $V = \int_a^b \pi y^2\, dx$ (M1)

Volume = $\int_0^{\ln 2} \pi(1+e^{2x})^2\, dx \quad \left(\text{or} \int_0^{0.805} \pi(1+e^{2x})^2\, dx\right)$ A2 3

[14]

53.) (a) $f'(x) = 5(3x+4)^4 \times 3 \;\left(=15(3x+4)^4\right)$ (A1)(A1)(A1) (C3)

(b) $\int (3x+4)^5 dx = \frac{1}{3}\times\frac{1}{6}(3x+4)^6 + c \left(=\frac{(3x+4)^6}{18}+c\right)$ (A1)(A1)(A1) (C3)

[6]

54.) Attempting to integrate. (M1)

$y = x^3 - 5x + c$ (A1)(A1)(A1)

substitute (2, 6) to find c $\left(6 = 2^3 - 5(2) + c\right)$ (M1)

$c = 8$ (A1)

$y = x^3 - 5x + 8$ (Accept $x^3 - 5x + 8$) (C6)

[6]

55.) (a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x) + g(x)\right) = f'(x) + g'(x) \quad \left(= f'(4) + g'(4)\right)$ (M1)

$= 7 + 4$

$= 11$ (A1) (C2)

(b) $\int_1^3 \left(g'(x) + 6\right)\mathrm{d}x = \left[g(x)\right]_1^3 + \left[6x\right]_1^3$ (A1)(A1)

$= \left(g(3) - g(1)\right) + \left(18 - 6\right) \left(= (2 - 1) + 12\right)$ (A1)

$= 13$ (A1) (C4)

[6]

56.) Using $\int \frac{1}{x} = \ln x$ (may be implied) (M1)

$\int_3^k \frac{1}{x-2}\,\mathrm{d}x = [\ln(x-2)]_3^k$ (A1)

$= \ln(k - 2) - \ln 1$ (A1)(A1)

$\ln(k - 2) - \ln 1 = \ln 7$

$k - 2 = 7$ (A1)

$k = 9$ (A1) (C6)

[6]

57.) (a) $s = 25t - \frac{4}{3}t^3 + c$ (M1)(A1)(A1)

***Note**: Award no further marks if "c" is missing.*

Substituting $s = 10$ and $t = 3$ (M1)

$10 = 25 \times 3 - \frac{4}{3}(3)^3 + c$

$10 = 75 - 36 + c$

$c = -29$ (A1)

$s = 25t - \frac{4}{3}t^3 - 29$ (A1) (N3)

(b) **METHOD 1**

s is a maximum when $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 0$ (may be implied) (M1)

$25 - 4t^2 = 0$ (A1)

$t^2 = \frac{25}{4}$

$t = \frac{5}{2}$ (A1) (N2)

METHOD 2

Using maximum of s ($12\frac{2}{3}$, may be implied) (M1)

$25t - \frac{4}{3}t^3 - 29 = 12\frac{2}{3}$ (A1)

t = 2.5 (A1) (N2)

(c) $25t - \frac{4}{3}t^3 - 29 > 0$ (accept equation) (M1)

$m = 1.27$, $n = 3.55$ (A1)(A1) (N3)

[12]

58.)

Note: *There are many approaches possible. However, there must be some evidence of their method.*

Area = $\int_0^k \sin 2x\,dx$ (must be seen somewhere) (A1)

Using area = 0.85 (must be seen somewhere) (M1)

EITHER

Integrating $\left[\frac{-1}{2}\cos 2x\right]_0^k$

$\left(= \frac{-1}{2}\cos 2k + \frac{1}{2}\cos 0\right)$ (A1)

Simplifying $\frac{-1}{2}\cos 2k + 0.5$ (A1)

Equation $\frac{-1}{2}\cos 2k + 0.5 = 0.85$ ($\cos 2k = -0.7$)

OR

Evidence of using trial and error on a GDC (M1)(A1)

Eg $\int_0^{\frac{\pi}{2}} \sin 2x\,dx = 0.5$, $\frac{\pi}{2}$ too small etc

OR

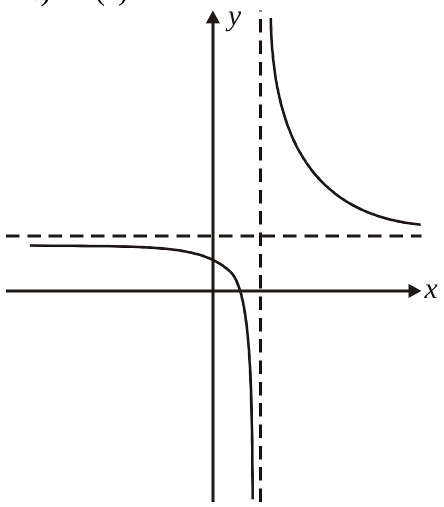
Using GDC and solver, starting with $\int_0^k \sin 2x\,dx - 0.85 = 0$ (M1)(A1)

THEN

$k = 1.17$ (A2) (N3)

[6]

59.) (a)



(A1)(A1) 2

Note: *Award (A1) for a second branch in approximately the correct position, and (A1) for the second branch having positive x and y intercepts. Asymptotes need not be drawn.*

(b) (i) x-intercept = $\frac{1}{2}\left(\text{Accept}\left(\frac{1}{2},0\right), x=\frac{1}{2}\right)$ (A1)

y-intercept = 1 (Accept (0, 1), y = 1) (A1)

(ii) horizontal asymptote $y = 2$ (A1)

vertical asymptote $x = 1$ (A1) 4

(c) (i) $f'(x) = 0 - (x-1)^{-2}\left(= \frac{-1}{(x-1)^2}\right)$ (A2)

(ii) no maximum / minimum points.

since $\frac{-1}{(x-1)^2} \neq 0$ (R1) 3

(d) (i) $2x + \ln(x-1) + c$ (accept $\ln|x-1|$)(A1)(A1)(A1)

(ii) $A = \int_2^4 f(x)\,dx\left(\text{Accept}\int_2^4\left(2+\frac{1}{x-1}\right)dx, \left[2x+\ln(x-1)\right]_2^4\right)$ (M1)(A1)

Notes: *Award (A1) for* ***both*** *correct limits.*
Award (M0)(A0) for an incorrect function.

(iii) $A = \left[2x+\ln(x-1)\right]_2^4$

$= (8 + \ln 3) - (4 + \ln 1)$ (M1)

$= 4 + \ln 3 (= 5.10$, to 3 sf) (A1) (N2) 7

[16]

60.) $f(x) = -\frac{1}{2}e^{-2x} + \ln(1-x) + c$ (M1)(A1)(A1)

Substituting $4 = -\frac{1}{2}e^{-2(0)} + \ln(1-0) + c$ $\left(\text{or } 4 = -\frac{1}{2} + \ln 1 + c\right)$ (M1)

$c = 4.5$ (A1)

$f(x) = -\frac{1}{2}e^{-2x} + \ln(1-x) + 4.5$ (A1)(C2)(C2)(C2)

[6]

61.) (a) (i) 16 (A2) (C2)

(ii) $\int_0^3 f(x)\,dx + \int_0^3 2\,dx$ (or appropriate sketch) (M1)

$= 14$ (A1) (C2)

(b) $\int_c^d f(x-2)\,dx = 8$

$c = 2, d = 5$ (A2) (C2)

[6]

62.) (a) (i) $a = 1 - \pi$ (accept $(1 - \pi, 0)$) (A1)

(ii) $b = 1 + \pi$ (accept $(1 + \pi, 0)$) (A1) 2

(b) (i) $\int_{-2.14}^{1} h(x)\,dx - \int_1^2 h(x)\,dx$ (M1)(A1)(A1)

OR

$\int_{-2.14}^{1} h(x)\,dx + \left|\int_1^2 h(x)\,dx\right|$ (M1)(A1)(A1)

OR

$\int_{-2.14}^{1} h(x)\,dx + \int_2^1 h(x)\,dx$ (M1)(A1)(A1)

(ii) $5.141... - (-0.1585...)$

$= 5.30$ (A2) 5

(c) (i) $y = 0.973$ (A1)

(ii) $-0.240 < k < 0.973$ (A3) 4

[11]

63.) (a) $y = 0$ (A1) 1

(b) $f'(x) = \frac{-2x}{(1+x^2)^2}$ (A1)(A1)(A1) 3

(c) $\frac{6x^2 - 2}{(1+x^2)^3} = 0$ (or sketch of $f'(x)$ showing the maximum) (M1)

$6x^2 - 2 = 0$ (A1)

$x = \pm\sqrt{\frac{1}{3}}$ (A1)

$x = \frac{-1}{\sqrt{3}} (= -0.577)$ (A1) (N4) 4

(d) $\int_{-0.5}^{0.5} \frac{1}{1+x^2}\,\mathrm{d}x \left(= 2\int_{0}^{0.5} \frac{1}{1+x^2}\,\mathrm{d}x = 2\int_{-0.5}^{0} \frac{1}{1+x^2}\,\mathrm{d}x \right)$ (A1)(A1)2

[10]

64.) (a) $\frac{1}{2} \times 10 = 5$ (M1)(A1) (C2)

(b) $\int_{1}^{3} g(x)\,\mathrm{d}x + \int_{1}^{3} 4\,\mathrm{d}x$ (M1)

$\int_{1}^{3} 4\,\mathrm{d}x \left[4x\right]_{1}^{3}$ (A1)

$= 4 \times 2 = 8$ (A1)

$\int_{1}^{3} \left(g(x) + 4\right)\mathrm{d}x = 10 + 8 = 18$ (A1) (C4)

[6]

65.) (a) (i) When $t = 0$, $v = 50 + 50\mathrm{e}^{0}$ (A1)

$= 100\ \mathrm{m\ s}^{-1}$ (A1)

(ii) When $t = 4$, $v = 50 + 50\mathrm{e}^{-2}$ (A1)

$= 56.8\ \mathrm{m\ s}^{-1}$ (A1) 4

(b) $v = \frac{\mathrm{d}s}{\mathrm{d}t} \Rightarrow s = \int v\,\mathrm{d}t$

$\int_{0}^{4} \left(50 + 50\mathrm{e}^{-0.5t}\right)\mathrm{d}t$ (A1)(A1)(A1) 3

Note: *Award (A1) for each limit in the correct position and (A1) for the function.*

(c) Distance travelled in 4 seconds $= \int_{0}^{4} \left(50 + 50\mathrm{e}^{-0.5t}\right)\mathrm{d}t$

$= \left[50t - 100\mathrm{e}^{-0.5t}\right]_{0}^{4}$ (A1)

$= (200 - 100\mathrm{e}^{-2}) - (0 - 100\mathrm{e}^{0})$

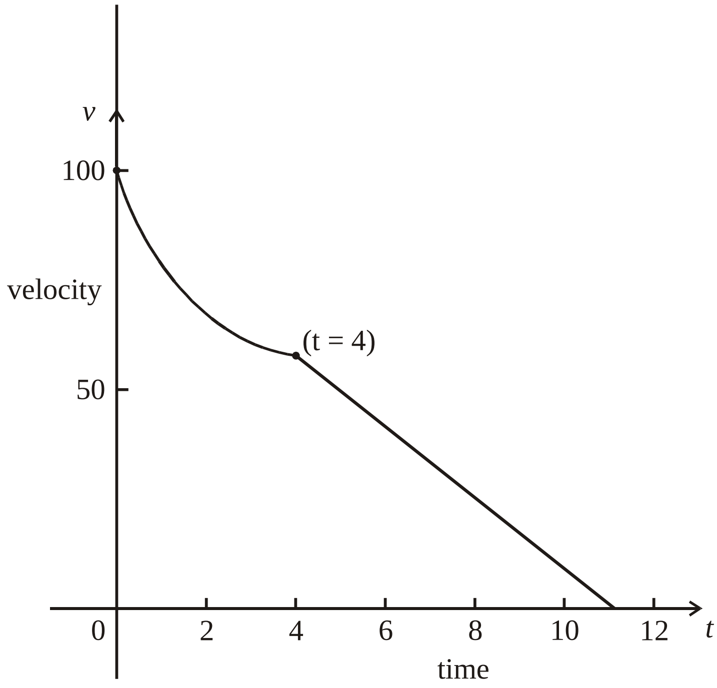
$= 286$ m (3 sf) (A1)

Note: *Award first (A1) for $[50t - 100e^{-0.5t}]$, ie limits not required.*

OR

Distance travelled in 4 seconds = 286 m (3 sf) (G2) 2

(d)



Notes: *Award (A1) for the exponential part, (A1) for the straight line through (11, 0),*
Award (A1) for indication of time on x-axis ***and*** *velocity on y-axis,*
(A1) for scale on x-axis ***and*** *y-axis.*
Award (A1) for marking the point where t = 4.

5

(e) Constant rate = $\frac{56.8}{7}$ (M1)

$= 8.11 \text{ m s}^{-2}$ (A1) 2

Note: *Award (M1)(A0) for –8.11.*

(f) distance = $\frac{1}{2}(7)(56.8)$ (M1)

= 199 m (A1) 2

Note: *Do not award* ***ft*** *in parts (e) and (f) if candidate has not used a straight line for t = 4 to t = 11 or if they continue the exponential beyond t = 4.*

[18]

66.) (a) (i) $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ (A1)

therefore $\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right) = 0$ (AG)

(ii) $\cos x + \sin x = 0 \Rightarrow 1 + \tan x = 0$

$\Rightarrow \tan x = -1$ (M1)

$x = \frac{3\pi}{4}$ (A1)

__Note:__ Award (A0) for 2.36.

OR

$x = \frac{3\pi}{4}$ (G2) 3

(b) $y = e^x(\cos x + \sin x)$

$\frac{dy}{dx} = e^x(\cos x + \sin x) + e^x(-\sin x + \cos x)$ (M1)(A1)(A1) 3

$= 2e^x \cos x$

(c) $\frac{dy}{dx} = 0$ for a turning point $\Rightarrow 2e^x \cos x = 0$ (M1)

$\Rightarrow \cos x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{2} \Rightarrow a = \frac{\pi}{2}$ (A1)

$y = e^{\frac{\pi}{2}}(\cos \frac{\pi}{2} + \sin \frac{\pi}{2}) = e^{\frac{\pi}{2}}$

$b = e^{\frac{\pi}{2}}$ (A1) 4

__Note:__ Award (M1)(A1)(A0)(A0) for $a = 1.57$, $b = 4.81$.

(d) At D, $\frac{d^2y}{dx^2} = 0$ (M1)

$2e^x \cos x - 2e^x \sin x = 0$ (A1)

$2e^x (\cos x - \sin x) = 0$

$\Rightarrow \cos x - \sin x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{4}$ (A1)

$\Rightarrow y = e^{\frac{\pi}{4}}(\cos \frac{\pi}{4} + \sin \frac{\pi}{4})$ (A1)

$= \sqrt{2}\, e^{\frac{\pi}{4}}$ (AG) 5

(e) Required area $= \int_0^{\frac{3}{4}} e^x (\cos x + \sin x)dx$ (M1)

$= 7.46$ sq units (G1)

OR

Area = 7.46 sq units (G2) 2

__Note__: Award (M1)(G0) for the answer 9.81 obtained if the calculator is in degree mode.

[17]

67.) $y = \int \frac{dy}{dx} dx$ (M1)

$= \frac{x^4}{4} + \frac{2x^2}{2} - x + c$ (A1)(A1)

__Note__: Award (A1) for first 3 terms, (A1) for "+ c".

$13 = \frac{16}{4} + 4 - 2 + c$ (M1)

$c = 7$ (A1)

$y = \dfrac{x^4}{4} + x^2 - x - 7$ (A1) (C6)

[6]

68.) (a) $\int (1 + 3\sin(x+2))\,dx = x - 3\cos(x+2) + c$ (A1)(A1)(A1) (C3)

Notes: *Award A1 for x, A1 for –cos (x + 2) A1 for coefficient 3, ie A1 A1 for the second term, which may be written as +3(–cos (x + 2))*
*Do **not** penalize the omission of c.*

(b) $1 + 3\sin(x + 2) = 0$ (M1)

$\sin(x + 2) = -\dfrac{1}{3}$

$x + 2 = -0.3398, \pi + 0.3398, \ldots$ (A1)

$x = -2.3398, 1.4814, \ldots$

Required value of $x = 1.48$ (A1) (C3)

[6]

69.) (a) (i) $f'(x) = -2e^{-2x}$ (A1)

(ii) $f'(x)$ is always negative (R1) 2

(b) (i) $y = 1 + e^{-2\times -\frac{1}{2}}$ (= 1 + e) (A1)

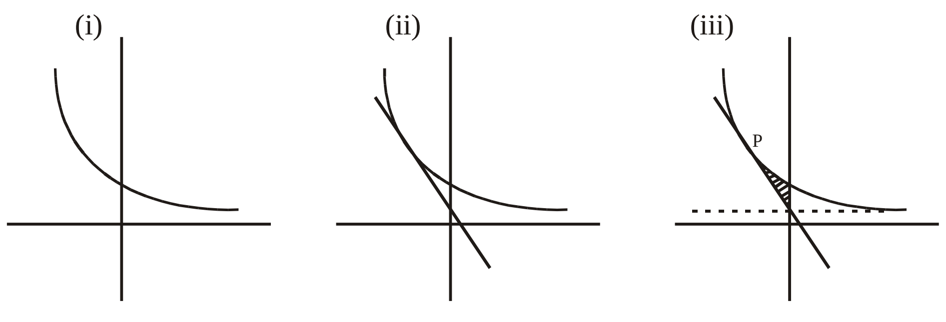
(ii) $f'\left(-\dfrac{1}{2}\right) = -2e^{-2\times -\frac{1}{2}}$ (= –2e) (A1) 2

Note: *In part (b) the answers do not need to be simplified.*

(c) $y - (1 + e) = -2e\left(x + \dfrac{1}{2}\right)$ (M1)

$y = -2ex + 1$ ($y = -5.44x + 1$) (A1)(A1) 3

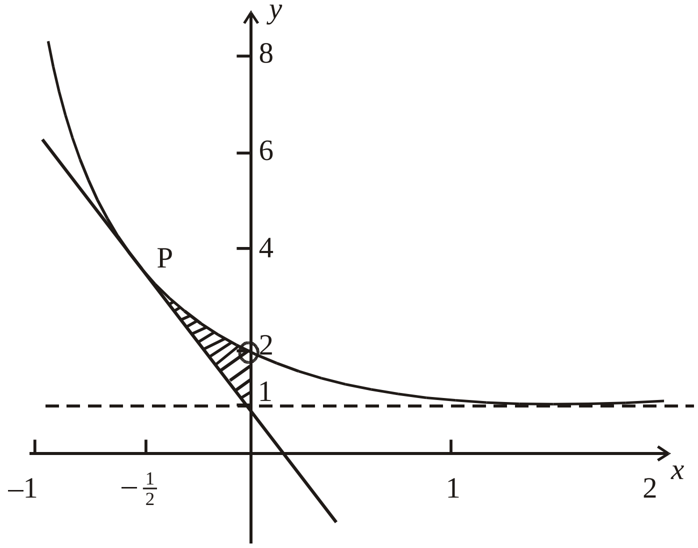
(d)



(A1)(A1)(A1)

Notes: *Award (A1) for each correct answer. Do **not** allow **(ft)** on an incorrect answer to part (i). The correct final diagram is shown below. Do not penalize if the horizontal asymptote is missing. Axes do not need to be labelled.*

(i)(ii)(iii)



(iv) Area = $\int_{-\frac{1}{2}}^{0} [(1 + e^{-2x}) - (-2ex + 1)]dx$ (or equivalent) (M1)(M1)

Notes: *Award (M1) for the limits, (M1) for the function.*
Accept difference of integrals as well as integral of difference.
Area below line may be calculated geometrically.

Area = $\int_{-\frac{1}{2}}^{0} [(e^{-2x} + 2ex)dx$

$= \left[-\frac{1}{2}e^{-2x} + ex^2\right]_{-\frac{1}{2}}^{0}$ (A1)

= 0.1795 …= 0.180 (3 sf) (A1)

OR

Area = 0.180 (G2) 7

[14]

70.) $f(x) = \int\left(\frac{1}{x+1} - 0.5\sin x\right)dx$ (M1)

$= \ln|x + 1| + 0.5\cos x + c$ (A1)(A1)(A1)

$2 = \ln 1 + 0.5 + c$ (M1)

$c = 1.5$ (A1)

$f(x) = \ln|x + 1| + 0.5\cos x + 1.5$ (C6)

[6]

71.) (a) $\int_0^1 e^{-kx}\,dx = \left[-\frac{1}{k}e^{-kx}\right]_0^1$ (A1)

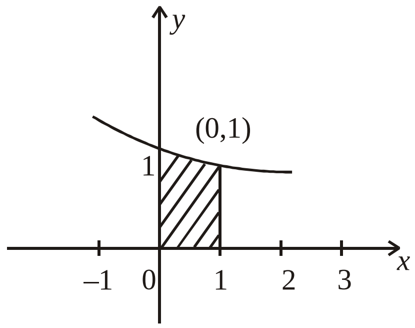
$= -\frac{1}{k}(e^{-k} - e^0)$ (A1)

$= -\frac{1}{k}(e^{-k} - 1)$ (A1)

$= -\frac{1}{k}(1 - e^{-k})$ (AG) 3

(b) $k = 0.5$

(i)



(A2)

Note: *Award (A1) for shape, and (A1) for the point (0,1).*

(ii) Shading (see graph) (A1)

(iii) Area = $\int_0^1 e^{-kx}\,dx$ for k = 0.5 (M1)

$= \frac{1}{0.5}(1 - e^{0.5})$

= 0.787 (3 sf) (A1)

OR

Area = 0.787 (3 sf) (G2) 5

(c) (i) $\frac{dy}{dx} = -ke^{-kx}$ (A1)

(ii) $x = 1$ $y = 0.8$ $\Rightarrow 0.8 = e^{-k}$ (A1)

$\ln 0.8 = -k$

$k = 0.223$ (A1)

(iii) At $x = 1$ $\frac{dy}{dx} = -0.223e^{-0.223}$ (M1)

= –0.179 (accept –0.178) (A1)

OR

$\frac{dy}{dx} = -0.178$ or – 0.179 (G2) 5

[13]

72.) $f(x) = x^{\frac{3}{2}}$ (M1)

(a) $f'(x) = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}}$ (or $\frac{3}{2}\sqrt{x}$) (M1)(A1) (C3)

(b) $\int x^{\frac{3}{2}}dx = \frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1} + c$ (M1)

$= \frac{2}{5}x^{\frac{5}{2}} + c$ (or $\frac{2}{5}\sqrt{x^5} + c$) (A1)(A1) (C3)

Notes: *Do not penalize the absence of c.*

Award (A1) for $\frac{5}{2}$ *and (A1) for* $x^{\frac{5}{2}}$.

[6]

73.) Area $= \int_a^b \sin x \, dx$ (M1)

$a = 0, b = \frac{3\pi}{4}$ (A1)

Area $= \int_0^{\frac{3\pi}{4}} \sin x \, dx = [-\cos x]_0^{\frac{3\pi}{4}}$ (A1)

$= \left(-\cos\frac{3\pi}{4}\right) - (-\cos 0)$ (A1)

$= -\left(-\frac{\sqrt{2}}{2}\right) - (-1)$ (A1)

$= 1 + \frac{\sqrt{2}}{2}$ (A1) (C6)

Note: *Award (G3) for a gdc answer of 1.71 or 1.707.*

[6]

74.) (a) At A, $x = 0$ => $y = \sin(e^0) = \sin(1)$ (M1)

=> coordinates of A = (0,0.841) (A1)

OR

A(0, 0.841) (G2) 2

(b) $\sin(e^x) = 0$ => $e^x = \pi$ (M1)

=> $x = \ln \pi$ (or $k = \pi$) (A1)

OR

$x = \ln \pi$ (or $k = \pi$) (A2) 2

(c) (i) Maximum value of sin function = 1 (A1)

(ii) $\frac{dy}{dx} = e^x \cos(e^x)$ (A1)(A1)

Note: *Award (A1) for* $\cos(e^x)$ *and (A1) for* e^x.

(iii) $\frac{dy}{dx} = 0$ at a maximum (R1)

$e^x \cos(e^x) = 0$

=> $e^x = 0$ (impossible) or $\cos(e^x) = 0$ (M1)

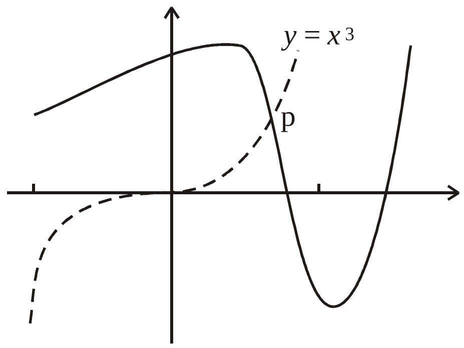
=> $e^x = \frac{\pi}{2}$ => $x = \ln\frac{\pi}{2}$ (A1)(AG) 6

(d) (i) Area $= \int_0^{\ln \pi} \sin(e^x) \, dx$ (A1)(A1)(A1)

Note: *Award (A1) for 0, (A1) for ln π, (A1) for sin (e^x).*

(ii) Integral = 0.90585 = 0.906 (3 sf) (G2) 5

(e)



(M1)

At P, x = 0.87656 = 0.877 (3 sf) (G2) 3

[18]

75.) (a) $\frac{ds}{dt} = 30 - at \Rightarrow s = 30t - a\frac{t^2}{2} + C$ (A1)(A1)(A1)

Note: *Award (A1) for 30t, (A1) for $a\frac{t^2}{2}$, (A1) for C.*

$t = 0 \Rightarrow s = 30(0) - a\frac{(0^2)}{2} + C = 0 + C \Rightarrow C = 0$ (M1)

$\Rightarrow s = 30t - \frac{1}{2}at^2$ (A1) 5

(b) (i) vel = 30 – 5(0) = 30 m s^{-1} (A1)

(ii) Train will stop when $0 = 30 - 5t \Rightarrow t = 6$ (M1)

Distance travelled = $30t - \frac{1}{2}at^2$

$= 30(6) - \frac{1}{2}(5)(6^2)$ (M1)

= 90m (A1)

90 < 200 => train stops before station. (R1)(AG) 5

(c) (i) $0 = 30 - at \Rightarrow t = \frac{30}{a}$ (A1)

(ii) $30\left(\frac{30}{a}\right) - \frac{1}{2}(a)\left(\frac{30}{a}\right)^2 = 200$ (M1)(M1)

Note: *Award (M1) for substituting $\frac{30}{a}$, (Ml) for setting equal to 200.*

$\Rightarrow \frac{900}{a} - \frac{450}{a} = \frac{450}{a} = 200$ (A1)

$\Rightarrow a = \frac{450}{200} = \frac{9}{4} = 2.25$ m s $^{-2}$ (A1) 5

Note: *Do not penalize lack of units in answers.*

[15]

76.) ***Note:*** *Do not penalize for the omission of C.*

(a) $\int \sin(3x+7)\mathrm{d}x = -\frac{1}{3}\cos(3x+7) + C$ (A1)(A1) (C2)

Note: *Award (A1) for* $\frac{1}{3}$*, (A1) for –cos (3x + 7).*

(b) $\int e^{-4x}\mathrm{d}x = -\frac{1}{4}e^{-4x} + C$ (A1)(A1) (C2)

Note: *Award (A1) for* $-\frac{1}{4}$*, (A1) for* e^{-4x}.

[4]

77.) (a) (i) $a = -3$ (A1)

(ii) $b = 5$ (A1) 2

(b) (i) $f'(x) = -3x^2 + 4x + 15$ (A2)

(ii) $-3x^2 + 4x + 15 = 0$

$-(3x + 5)(x - 3) = 0$ (M1)

$x = -\frac{5}{3}$ or $x = 3$ (A1)(A1)

OR

$x = -\frac{5}{3}$ or $x = 3$ (G3)

(iii) $x = 3 \Rightarrow f(3) = -3^3 + 2(3^2) + 15(3)$ (M1)

$= -27 + 18 + 45 = 36$ (A1)

OR

$f(3) = 36$ (G2) 7

(c) (i) $f'(x) = 15$ at $x = 0$ (M1)

Line through (0, 0) of gradient 15

$\Rightarrow y = 15x$ (A1)

OR

$y = 15x$ (G2)

(ii) $-x^3 + 2x^2 + 15x = 15x$ (M1)

$\Rightarrow -x^3 + 2x^2 = 0$

$\Rightarrow -x^2(x - 2) = 0$

$\Rightarrow x = 2$ (A1)

OR

$x = 2$ (G2) 4

(d) Area =115 (3 sf) (G2)

OR

Area = $\int_0^6 (-x^3 + 2x^2 + 15x)\mathrm{d}x = \left[-\frac{x^4}{4} + 2\frac{x^3}{3} + 15\frac{x^2}{2}\right]_0^5$ (M1)

$= \frac{1375}{12} = 115$ (3 sf) (A1) 2

[15]

78.) (a) (i) $v(0) = 50 - 50e^0 = 0$ (A1)

(ii) $v(10) = 50 - 50e^{-2} = 43.2$ (A1) 2

(b) (i) $a = \frac{\mathrm{d}v}{\mathrm{d}t} = -50(-0.2e^{-0.2t})$ (M1)

$= 10e^{-0.2t}$ (A1)

(ii) $a(0) = 10e^0 = 10$ (A1) 3

(c) (i) $t \to \infty \Rightarrow v \to 50$ (A1)

(ii) $t \to \infty \Rightarrow a \to 0$ (A1)

(iii) when $a = 0$, v is constant at 50 (R1) 3

(d) (i) $y = \int v\mathrm{d}t$ (M1)

$= 50t - \frac{e^{-0.2t}}{-0.2} + k$ (A1)

$= 50t + 250e^{-0.2t} + k$ (AG)

(ii) $0 = 50(0) + 250e^0 + k = 250 + k$ (M1)

$\Rightarrow k = -250$ (A1)

(iii) Solve $250 = 50t + 250e^{-0.2t} - 250$ (M1)

$\Rightarrow 50t + 250e^{-0.2t} - 500 = 0$

$\Rightarrow t + 5e^{-0.2t} - 10 = 0$

$\Rightarrow t = 9.207\ s$ (G2) 7

[15]

79.) $f'(x) = 1 - x^2$

$f(x) = \int (1 - x^2)\mathrm{d}x = x - \frac{x^3}{3} + C$ (A1)

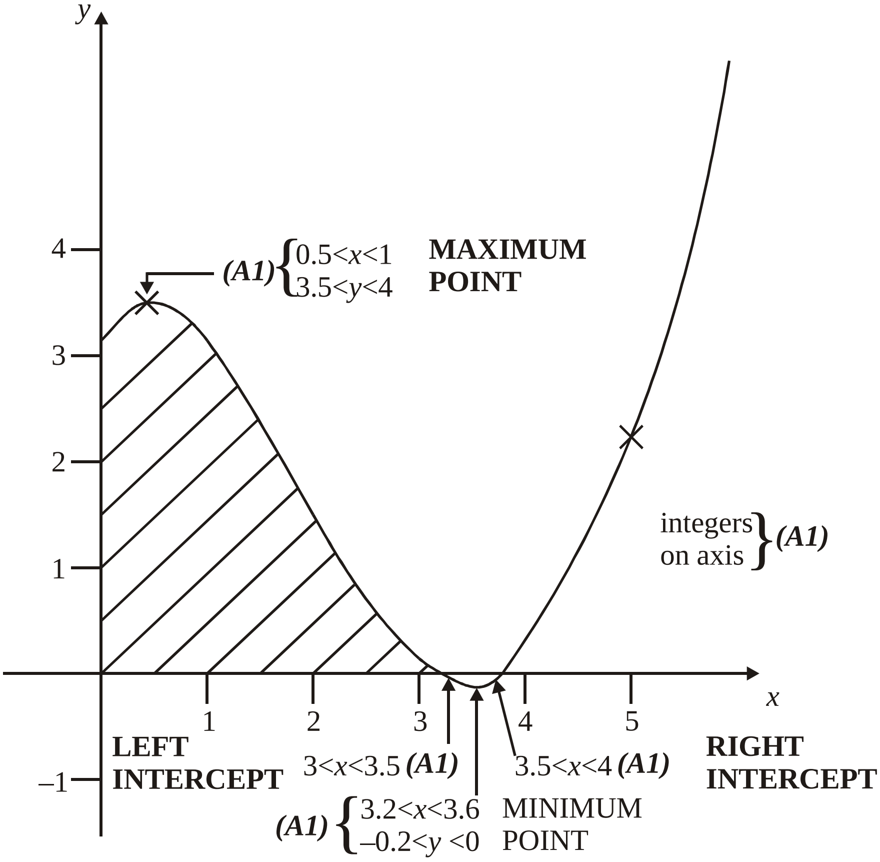
$f(3) = 0 \Rightarrow 3 - 9 + C = 0$ (M1)

$\Rightarrow c = 6$ (A1)

$f(x) = x - \frac{x^3}{3} + 6$ (A1)

[4]

80.) (a)



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(b) π is a solution if and only if $\pi + \pi\cos\pi = 0$. (M1)
Now $\pi + \pi\cos\pi = \pi + \pi(-1)$ (A1)
$= 0$ (A1) 3

(c) By using appropriate calculator functions $x = 3.696\,722\,9...$ (M1)
$\Rightarrow x = 3.69672$ (6sf) (A1) 2

(d) See graph: (A1)
$\int_0^\pi (\pi + x\cos x)\,dx$ (A1) 2

(e) **EITHER** $\int_0^\pi (\pi + x\cos x)\,dx = 7.86960$ (6 sf) (A3) 3

Note: *This answer assumes appropriate use of a calculator eg*

'fnInt': $\begin{cases} fnInt(Y_1, X, 0, \pi) = 7.869604401 \\ with\, Y_1 = \pi + x\cos x \end{cases}$

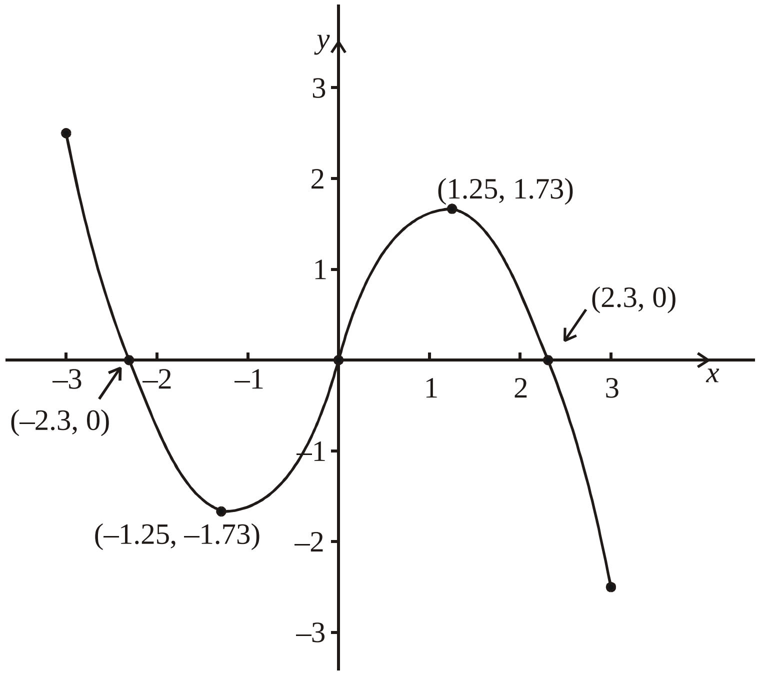
OR $\int_0^\pi (\pi + x\cos x)\,dx = [\pi x + x\sin x + \cos x]_0^\pi$
$= \pi(\pi - 0) + (\pi\sin\pi - 0 \times \sin 0) + (\cos\pi - \cos 0)$ (A1)
$= \pi^2 + 0 + -2 = 7.86960$ (6 sf) (A1) 3

[15]

81.) $f'(x) = \cos x \Rightarrow f(x) = \sin x + C$ (M1)
$f\left(\frac{\pi}{2}\right) = -2 \Rightarrow -2 = \sin\left(\frac{\pi}{2}\right) + C$ (M1)
$C = -3$ (A1)
$f(x) = \sin x - 3$ (A1) (C4)

[4]

82.) (a) $y = \pi \sin x - x$



(A5) 5

Notes: *Award (A1) for appropriate scales marked on the axes.*
Award (A1) for the x-intercepts at (ï2.3, 0).
Award (A1) for the maximum and minimum points at (ï1.25, ï1.73).
Award (A1) for the end points at (ï3, ï2.55).
Award (A1) for a smooth curve.
Allow some flexibility, especially in the middle three marks here.

(b) $x = 2.31$ (A1) 1

(c) $\int (\pi \sin x - x)\mathrm{d}x = -\pi \cos x - \frac{x^2}{2} + C$ (A1)(A1)

Note: *Do not penalize for the absence of C.*

Required area = $\int_0^1 (\pi \sin x - x)\mathrm{d}x$ (M1)

$= 0.944$ (G1)

OR area = 0.944 (G2) 4

[10]

83.) $f'(x) = -2x + 3$

$f(x) = \frac{-2x^2}{2} + 3x + c$ (M1)

Notes: *Award (M1) for an attempt to integrate. Do not penalize the omission of c here.*

$1 = -1 + 3 + c$ (A1)

$c = -1$ (A1)

$f(x) = -x^2 + 3x - 1$ (A1) (C4)

[4]

84.) (a) $f'(x) = 3(2x + 5)^2 \times 2$ (M1)(A1)

Note: *Award (M1) for an attempt to use the chain rule.*

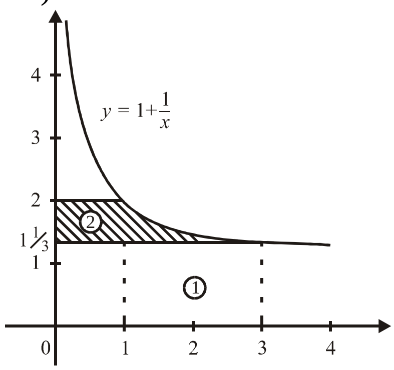
$= 6(2x + 5)^2$ (C2)

(b) $\int f(x)\mathrm{d}x = \frac{(2x+5)^4}{4\times 2} + c$ (A2) (C2)

Note: *Award (A1) for $(2x + 5)^4$ and (A1) for /8.*

[4]

85.)



Area = $\int_{1\frac{1}{3}}^{2} x\mathrm{d}y = \int_{1\frac{1}{3}}^{2} \frac{1}{(y-1)}\mathrm{d}y$ (M1)(A1)

$= [\ln(y-1)]_{1\frac{1}{3}}^{2}$

$= \ln 1 - \ln\frac{1}{3}$ (A1)

$= \ln 3$ (A1) (C4)

OR

Area from $x = 1$ to $x = 3$, $A = \int_1^3 \left(1+\frac{1}{x}\right)\mathrm{d}x = [x + \ln x]_1^3$

$= (3 + \ln 3) - (1 + \ln 1)$ (M1)

$= 2 + \ln 3$ (A1)

Area rectangle ① $= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$, area rectangle ② $= 1 \times \frac{2}{3} = \frac{2}{3}$

Shaded area $= 2 + \ln 3 - 2\frac{2}{3} + \frac{2}{3}$ (M1)

$= \ln 3$ (A1) (C4)

OR

Area from $x = 1$ to $x = 3$, $A = \int_1^3 \left(1+\frac{1}{x}\right)\mathrm{d}x$ (M1)

$A = 3.0986 \ldots$ (G0)

Area rectangle ① $= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$, area rectangle ② $= 1 \times \frac{2}{3} = \frac{2}{3}$

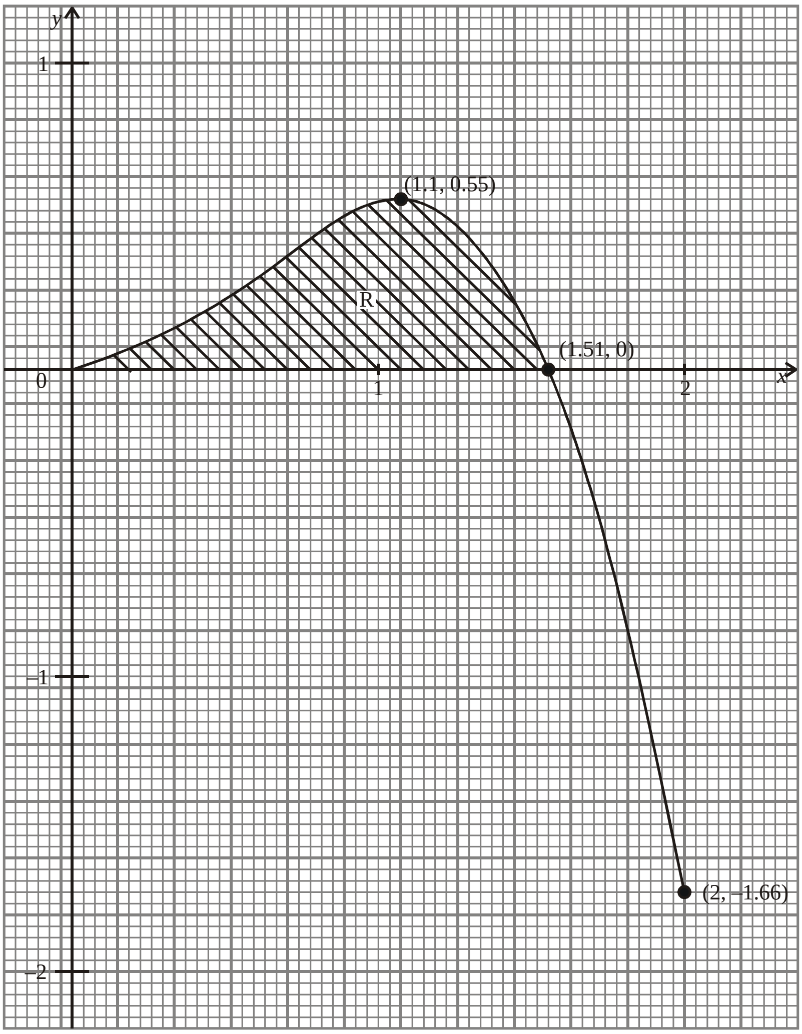
Shaded area = $3.0986 - 2\left(\frac{2}{3} + \frac{2}{3}\right)$ (M1)

$= 1.10$ (3 sf) (A1) (C4)

Notes: *An exact value is required. If candidates have obtained the answer 1.10, and shown their working, award marks as above. However, if they do not show their working, award (G2) for the correct answer of 1.10.*
Award no marks for the giving of 3.10 as the final answer.

[4]

86.) (a)(i) & (c)(i)



(A3)

Notes: *The sketch does* **not** *need to be on graph paper. It should have the correct shape, and the points (0, 0), (1.1, 0.55), (1.57, 0) and (2, –1.66) should be indicated in some way.*
Award (A1) for the correct shape.
Award (A2) for 3 or 4 correctly indicated points, (A1) for 1 or 2 points.

(ii) Approximate positions are
positive x-intercept (1.57, 0) (A1)
maximum point (1.1, 0.55) (A1)
end points (0, 0) and (2, –1.66) (A1)(A1) 7

(b) $x^2 \cos x = 0$ $x \neq 0 \Rightarrow \cos x = 0$ (M1)

$\Rightarrow x = \dfrac{\pi}{2}$ (A1) 2

Note: *Award (A2) if answer correct.*

(c) (i) see graph (A1)

(ii) $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$ (A2) 3

Note: *Award (A1) for limits, (A1) for rest of integral correct (do not penalize missing dx).*

(d) Integral = 0.467 (G3)

OR

Integral = $\left[x^2 \sin x + 2x\cos x - 2\sin x\right]_0^{\pi/2}$ (M1)

$= \left[\frac{\pi^2}{4}(1) + 2\left(\frac{\pi}{2}\right)(0) - 2(1)\right] - [0 + 0 - 0]$ (M1)

$= \frac{\pi}{2} - 2$ (exact) **or** 0.467 (3 sf) (A1) 3

[15]

87.) (a) From graph, period = 2π (A1) 1

(b) Range = $\{y \mid -0.4 < y < 0.4\}$ (A1) 1

(c) (i) $f'(x) = \frac{d}{dx}\{\cos x\,(\sin x)^2\}$

$= \cos x\,(2\sin x\cos x) - \sin x\,(\sin x)^2$ **or** $-3\sin^3 x + 2\sin x$(M1)(A1)(A1)

Note: *Award (M1) for using the product rule and (A1) for each part.*

(ii) $f'(x) = 0$ (M1)

$\Rightarrow \sin x\{2\cos x - \sin^2 x\} = 0$ **or** $\sin x\{3\cos x - 1\} = 0$ (A1)

$\Rightarrow 3\cos^2 x - 1 = 0$

$\Rightarrow \cos x = \pm\sqrt{\left(\frac{1}{3}\right)}$ (A1)

At A, $f(x) > 0$, hence $\cos x = \sqrt{\left(\frac{1}{3}\right)}$ (R1)(AG)

(iii) $f(x) = \sqrt{\left(\frac{1}{3}\right)}\left(1 - \left(\sqrt{\left(\frac{1}{3}\right)}\right)^2\right)$ (M1)

$= \frac{2}{3} \times \frac{1}{\sqrt{3}} = \frac{2}{9}\sqrt{3}$ (A1) 9

(d) $x = \frac{\pi}{2}$ (A1) 1

(e) (i) $\int (\cos x)(\sin x)^2\,dx = \frac{1}{3}\sin^3 x + c$ (M1)(A1)

(ii) Area = $\int_0^{\pi/2} (\cos x)(\sin x)^2\,dx = \frac{1}{3}\left\{\left(\sin\frac{\pi}{2}\right)^3 - (\sin 0)^3\right\}$ (M1)

$= \frac{1}{3}$ (A1) 4

(f) At C $f''(x) = 0$ (M1)

$\Leftrightarrow 9\cos^3 x - 7\cos x = 0$

$\Leftrightarrow \cos x(9\cos^2 x - 7) = 0$ (M1)

$\Rightarrow x = \frac{\pi}{2}$ (reject) ***or*** $x = \arccos\frac{\sqrt{7}}{3} = 0.491$ (3 sf) (A1)(A1) 4

[20]

88.) (a) $p = 3$ (A1) (C1)

(b) Area $= \int_0^{\frac{\pi}{2}} 3\cos x\,dx$ (M1)

$= [3\sin x]_0^{\frac{\pi}{2}}$ (A1)

= 3 square units (A1) (C3)

[4]

89.) (a) $f''(x) = 2x - 2$

$\Rightarrow$ $f'(x) = x^2 - 2x + c$ (M1)(M1)

$= 0$ when $x = 3$

$\Rightarrow$ $0 = 9 - 6 + c$

$c = -3$ (A1)

$f'(x) = x^2 - 2x - 3$ (AG)

$f(x) = \frac{x^3}{3} - x^2 - 3x + d$ (M1)

When $x = 3$, $f(x) = -7$

$\Rightarrow$ $-7 = 9 - 9 - 9 + d$ (M1)

$\Rightarrow$ $d = 2$ (A1) 6

$\Rightarrow$ $f(x) = \frac{x^3}{3} - x^2 - 3x + 2$

(b) $f(0) = 2$ (A1)

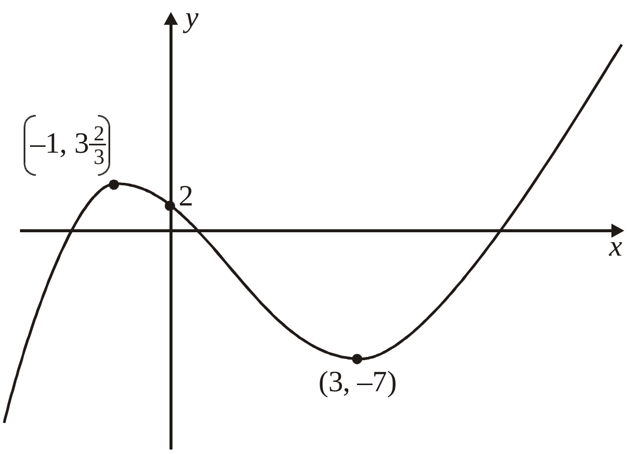
$f(-1) = -\frac{1}{3} - 1 + 3 + 2$

$= 3\frac{2}{3}$ (A1)

$f'(-1) = 1 + 2 - 3$

$= 0$ (A1) 3

(c) $f'(-1) = 0 \Rightarrow \left(-1, 3\frac{2}{3}\right)$ is a stationary point



(A4) 4

Note: *Award (A1) for maximum, (A1) for (0, 2) (A1) for (3, –7), (A1) for cubic.*

[13]

90.) (a) $y = e^{x/2}$ at $x = 0$ $y = e^0 = 1$ $P(0, 1)$ (A1)(A1) 2

(b) $V = \pi\int_0^{\ln 2}(e^{x/2})^2\,dx$ (A4) 4

Notes: *Award (A1) for* p
(A1) for each limit
(A1) for $(e^{x/2})^2$.

(c) $V = \int_0^{\ln 2} e^x dx$ (A1)

$= \pi[e^x]_0^{\ln 2}$ (A1)

$= \pi[e^{\ln 2} - e^0]$ (A1)

$= \pi[2 - 1] = \pi$ (A1)(A1)

$= \pi$ (AG) 5

[11]

91.) (a) $\int_0^1 12x^2\,(1-x)dx$ (A1) (C1)

(b) $12\int_0^1 (x^2 - x^3)\,dx$

$= 12\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$ (M1)

$= 12\left(\frac{1}{3} - \frac{1}{4}\right)$ (A1)

$= 1$ (A1) (C3)

[4]

92.) $\int_1^a \frac{1}{x}dx = 2$ (M1)

$\Rightarrow [\ln x]_1^a = 2$ (M1)

$\Rightarrow \ln a = 2$ (A1)

$\Rightarrow a = e^2$ (A1) (C4)

Note: *If 7.39 given instead of* e^2 *then deduct [1 mark].*

[4]

93.) (a) $y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ (A1)

when $x = e$, $\frac{dy}{dx} = \frac{1}{e}$

tangent line: $y = \left(\frac{1}{e}\right)(x - e) + 1$ (M1)

$y = \frac{1}{e}(x) - 1 + 1 = \frac{x}{e}$ (A1)

$x = 0 \Rightarrow y = \frac{0}{e} = 0$ (M1)

(0, 0) is on line (AG) 4

(b) $\frac{d}{dx}(x \ln x - x) = (1) \times \ln x + x \times \left(\frac{1}{x}\right) - 1 = \ln x$ (M1)(A1)(AG) 2

Note: *Award (M1) for applying the product rule, and (A1) for $(1) \times \ln x + x \times \left(\frac{1}{x}\right)$.*

(c) Area = area of triangle – area under curve (M1)

$= \left(\frac{1}{2} \times e \times 1\right) - \int_1^e \ln x dx$ (A1)

$= \frac{e}{2} - [x \ln x - x]_1^e$ (A1)

$= \frac{e}{2} - \{(e \ln e - 1 \ln 1) - (e - 1)\}$ (A1)

$= \frac{e}{2} - \{e - 0 - e + 1\}$

$= \frac{1}{2}e - 1.$ (AG) 4

[10]

94.) (a) $y = x(x - 4)_2$

(i) $y = 0 \Leftrightarrow x = 0$ or $x = 4$ (A1)

(ii) $\frac{dy}{dx} = 1(x - 4)^2 + x \times 2(x - 4) = (x - 4)(x - 4 + 2x)$

$= (x - 4)(3x - 4)$ (A1)

$\frac{dy}{dx} = 0 \Rightarrow x = 4$ or $x = \frac{4}{3}$ (A1)

$\left.\begin{array}{l} x = 1 \Rightarrow \frac{dy}{dx} = (-3)(-1) = 3 > 0 \\ x = 2 \Rightarrow \frac{dy}{dx} = (-2)(2) = -4 < 0 \end{array}\right\} \Rightarrow \frac{4}{3}$ is a **maximum** (R1)

Note: *A second derivative test may be used.*

$x = \frac{4}{3} \Rightarrow y = \frac{4}{3} \times \left(\frac{4}{3} - 4\right)^2 = \frac{4}{3} \times \left(\frac{-8}{3}\right)^2 = \frac{4}{3} \times \frac{64}{9} = \frac{256}{27}$

$\left(\frac{4}{3}, \frac{256}{27}\right)$ (A1)

Note: *Proving that $\left(\frac{4}{3}, \frac{256}{27}\right)$ is a maximum is not necessary to receive full credit of [4 marks] for this part.*

(iii) $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\big((x-4)(3x-4)\big) = \dfrac{d}{dx}(3x^2 - 16x + 16) = 6x - 16$ (A1)

$\dfrac{d^2y}{dx^2} = 0 \Leftrightarrow 6x - 16 = 0$ (M1)

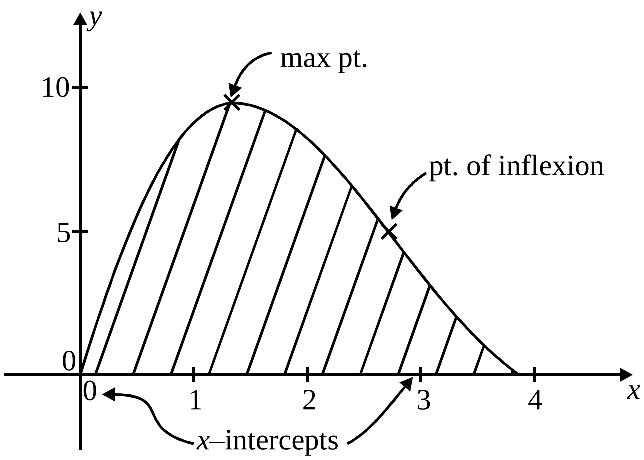
$\Leftrightarrow x = \dfrac{8}{3}$ (A1)

$x = \dfrac{8}{3} \Rightarrow y = \dfrac{8}{3}\left(\dfrac{8}{3} - 4\right)^2 = \dfrac{8}{3}\left(\dfrac{-4}{3}\right)^2 = \dfrac{8}{3} \times \dfrac{16}{9} = \dfrac{128}{27}$

$\left(\dfrac{8}{3}, \dfrac{128}{27}\right)$ (A1) 9

Note: *GDC use is likely to give the answer (1.33, 9.48). If this answer is given with no explanation, award (A2), If the answer is given with the explanation "used GDC" or equivalent, award full credit.*

(b)



(A3) 3

Note: *Award (A1) for intercepts, (A1) for maximum and (A1) for point of inflexion.*

(c) (i) See diagram above (A1)

(ii) $0 < y < 10$ for $0 \le x \le 4$ (R1)

So $\int_0^4 0\,dx < \int_0^4 y\,dx < \int_0^4 10\,dx \Rightarrow 0 < \int_0^4 y\,dx < 40$ (R1) 3

[15]